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Asymptotic properties of the Hahn–Hausman test for weak-instruments

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Abstract

This paper provides weak-instrument asymptotic representations of tests for instrument validity by Hahn and Hausman's (HH) [Hahn, J., Hausman, J., 2002. A new specification test for the validity of instrumental variables. Econometrica 70, 163–189.], and uses these representations to compute asymptotic power against weak or irrelevant instruments. The HH tests were proposed as pretests, and the asymptotic properties of post-test inferences, conditional on the tests failing to reject instrument validity, are also examined.

Keywords: Weak identification; Instrumental variables; Pretest

JEL classification: C12; C20

1. Introduction

Hahn and Hausman (2002; henceforth HH) recently proposed new tests for the validity of inferences based on conventional first-order asymptotics in overidentified instrumental variables (IV)

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regression. Consider the case of a single included endogenous regressor. If the instruments are valid, they reasoned, then standard first-order asymptotics implies that the two stage least squares (TSLS) estimator obtained by regressing one of the endogenous variables, y_1, on the other, y_2, should be close to the reciprocal of the TSLS estimator of the "reverse regression" of y_2 on y_1. Accordingly, HH propose a statistic that is the difference between the forward TSLS estimator and the reciprocal of the reverse TSLS estimator, adjusted for second-order bias and standardized by a second-order expression for the variance of this difference. They also propose a similarly motivated test statistic based on the Nagar (1959) — type bias adjusted TSLS (BTSLS) estimator of Donald and Newey (2001). Hahn and Hausman (2002, 2003) suggest that tests based on these statistics will reject if one or the other of the conditions for instrument validity fail, that is, if the instruments are weak and/or if they are endogenous.

This paper focuses on the use of a HH statistic to test the null hypothesis that instruments are strong against the alternative that they are weak. Although HH report Monte Carlo results, we are unaware of asymptotic results about the power or consistency of the HH tests against weak or irrelevant instruments. Accordingly, Section 2 provides the asymptotic distribution of the HH statistics

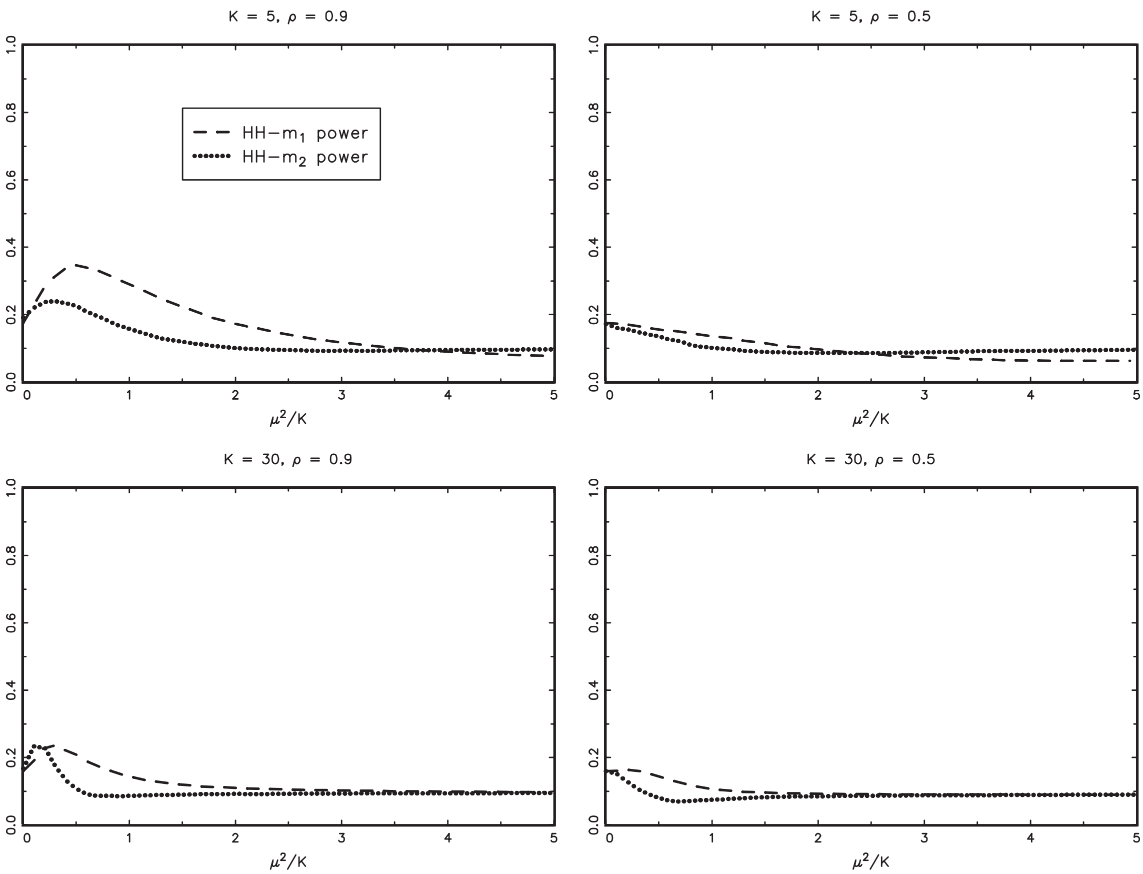

Fig. 1. Asymptotic power of 10% HH tests using m_1 and m_2 against weak-instruments, as a function of the concentration parameter divided by the number of instruments (μ^2/K).

for the case that sample is large but the instruments are weak or irrelevant. This entails applying the weak-instrument asymptotics of Staiger and Stock (1997), in which the so-called "concentration parameter," a unitless measure of the strength of the instruments and of the quality of the standard large-sample normal approximation (see Rothenberg, 1984), is held constant as the sample size increases. The HH tests were proposed as a pretest, and these weak-instrument limiting distributions, combined with results in Staiger and Stock (1997), provide asymptotic distributions of k-class estimators, conditional on passing a HH pretest.

Section 3 provides numerical results concerning the performance of the HH tests. First, we evaluate the asymptotic power of the HH tests against weak-instruments; next, we consider the performance of two post-test IV estimators, limited information maximum likelihood (LIML) and the estimator of Fuller (1977), conditional on passing a HH pretest. Section 3 also reports Monte Carlo results indicating that the weak-instrument asymptotics provide good approximations to the finite-sample distributions of interest when there are at least 100 observations.

We have three main findings. First, the asymptotic power of the HH tests against weak or irrelevant instruments is low and the tests are not consistent against nonidentification (irrelevant instruments); for

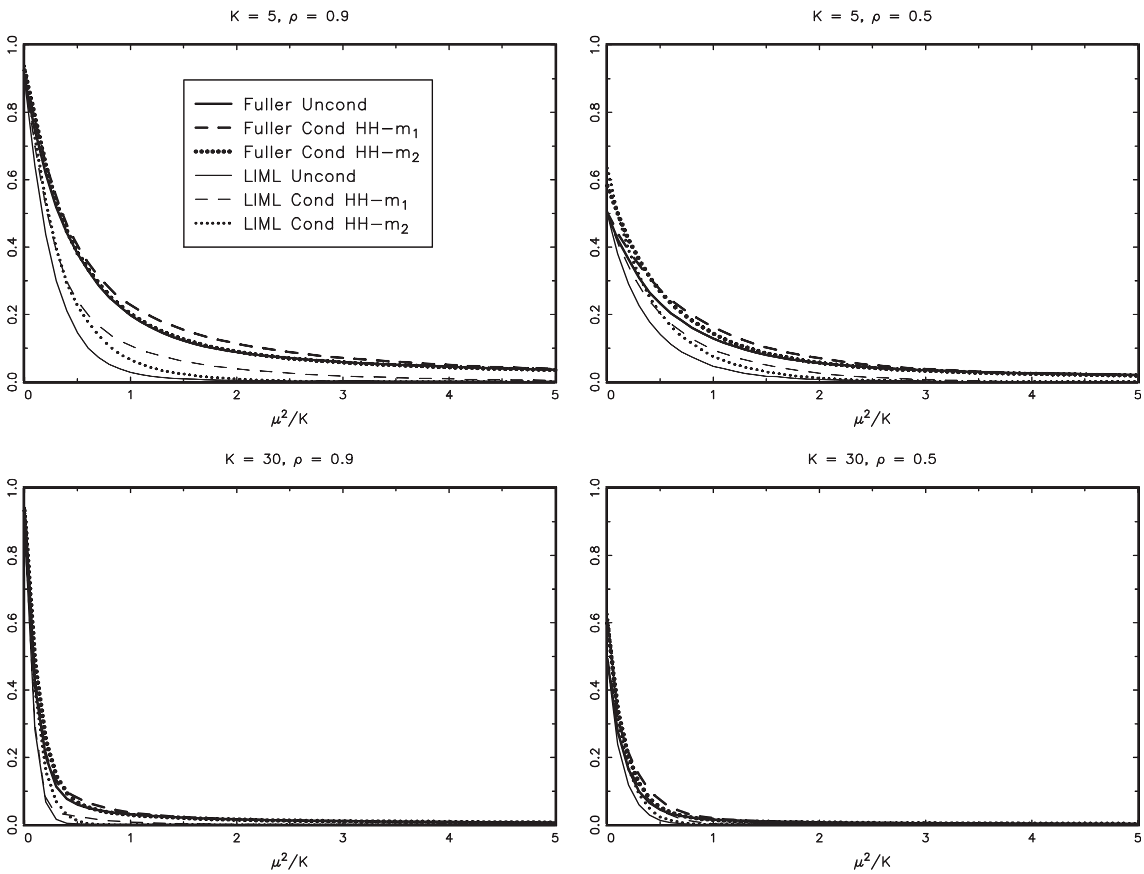

Fig. 2. Asymptotic median bias of the Fuller and LIML estimators: unconditional, conditional on $|m_1| \leq 1.645$ ("HH-m_1"), and conditional on $|m_2| \leq 1.645$ ("HH-m_2").

the parameter values we consider the power of a 10% HH test ranges from 8% to 35%. Second, a HH pretest tends to screen out large outliers for LIML but not for Fuller's estimator, although the pretest does not reduce the median bias of either estimator. Third, preliminary screening using a HH pretest neither reduces nor increases the size distortions of the LIML and Fuller Wald tests of $\beta=\beta_0$.

2. The HH test statistics and their weak-instrument asymptotic distributions

Following HH, consider the IV regression model with a single endogenous regressor:

$$\boldsymbol{y}_1 = \boldsymbol{y}_2\beta + \boldsymbol{u} \tag{1}$$

$$\boldsymbol{y}_2 = \mathbf{Z}\boldsymbol{\Pi} + \boldsymbol{v} \tag{2}$$

where $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are $n\times 1$ vectors of the n observations on the two endogenous variables, $\mathbf{Z}$ is a $n\times K$ matrix of observations on the K instrumental variables, β is the unknown scalar coefficient of interest, $\boldsymbol{\Pi}$

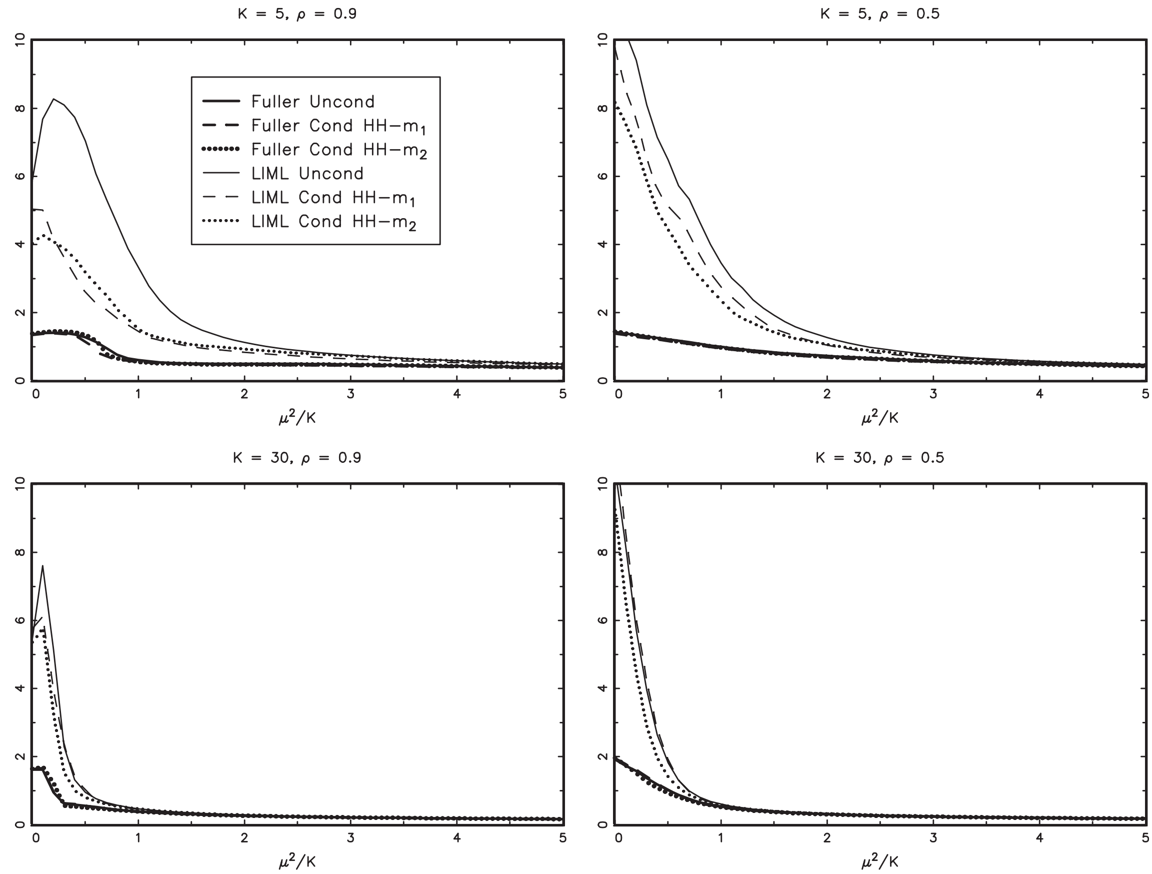

Fig. 3. 95% quantile of $|\hat{\beta}-\beta|$ for the Fuller and LIML estimators: unconditional, conditional on $|m_1|\leq 1.645$ ("HH-m_1"), and conditional on $|m_2|\leq 1.645$ ("HH-m_2").

is a $K \times 1$ unknown parameter vector, and $\boldsymbol{u}$ and $\boldsymbol{\nu}$ are $n \times 1$ vectors of i.i.d. errors with variances $\sigma_{\boldsymbol{u}}^2$ and $\sigma_{\boldsymbol{\nu}}^2$ and correlation ρ.

2.1. The HH test statistics

Let $\hat{\beta}$ denote an IV estimator of β, let $\hat{\sigma}_{\boldsymbol{u}}^2 = (\boldsymbol{y}_1 - \boldsymbol{y}_2\hat{\beta})'(\boldsymbol{y}_1 - \boldsymbol{y}_2\hat{\beta})/(n-1)$, and let $P_{\mathbf{Z}} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ and $M_{\mathbf{Z}} = \mathbf{I}_K - P_{\mathbf{Z}}$, where $\mathbf{I}_K$ is the $K \times K$ identity matrix. The HH TSLS-based test statistic is

$$m_1 = \hat{d}_1/\sqrt{\hat{w}_1}, \tag{3}$$

where[1]

$$\hat{d}_1 = \sqrt{n}\left[\frac{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1}{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2} - \frac{\boldsymbol{y}_1'P_{\mathbf{Z}}\boldsymbol{y}_1}{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1} + \frac{n^2\hat{\Xi}}{(\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2)(\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1)}\right]$$

$$\hat{\Xi} = \frac{K-1}{n^2}\left[\frac{\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_2}{n-K}\left(\boldsymbol{y}_1'P_{\mathbf{Z}}\boldsymbol{y}_1 - (K-1)\frac{\boldsymbol{y}_1'M_{\mathbf{Z}}\boldsymbol{y}_1}{n-K}\right)\right.$$
$$\left. - \frac{\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_1}{n-K}\left(2\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1 - (K-1)\frac{\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_1}{n-K}\right) + \frac{\boldsymbol{y}_1'M_{\mathbf{Z}}\boldsymbol{y}_1}{n-K}\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2\right]$$

$$\hat{w}_1 = \frac{2(K-1)(n-1)^2\hat{\sigma}_{\boldsymbol{u}}^4[\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2 - (K-1)\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_2/(n-K)]^2}{(n-K)(\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2)^2(\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1)^2}.$$

The HH Nagar-based test statistic is

$$m_2 = \hat{d}_2/\sqrt{\hat{w}_2}, \tag{4}$$

where

$$\hat{d}_2 = \sqrt{n}\left[\frac{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1 - (K-2)\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_1/(n-K+2)}{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2 - (K-2)\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_2/(n-K+2)}\right.$$
$$\left. - \frac{\boldsymbol{y}_1'P_{\mathbf{Z}}\boldsymbol{y}_1 - (K-2)\boldsymbol{y}_1'M_{\mathbf{Z}}\boldsymbol{y}_1/(n-K+2)}{\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_1 - (K-2)\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_1/(n-K+2)}\right]$$

$$\hat{w}_2 = \frac{2(K-1)(n-1)^2\hat{\sigma}_{\boldsymbol{u}}^4}{(n-K)\hat{\beta}^2[\boldsymbol{y}_2'P_{\mathbf{Z}}\boldsymbol{y}_2 - (K-1)\boldsymbol{y}_2'M_{\mathbf{Z}}\boldsymbol{y}_2/(n-K)]^2}.$$

Using second-order asymptotics under the assumption of strong instruments, HH show that m_1 and m_2 have standard normal null distributions.

[1] The expression for $\hat{\Xi}$ given here is obtained by substituting $\hat{\alpha}/(1-\hat{\alpha}) = (K-1)/(n-K)$, as used in HH Eq. (3.8), into the expression for $\hat{\Xi}$ following HH Eq. (3.5).

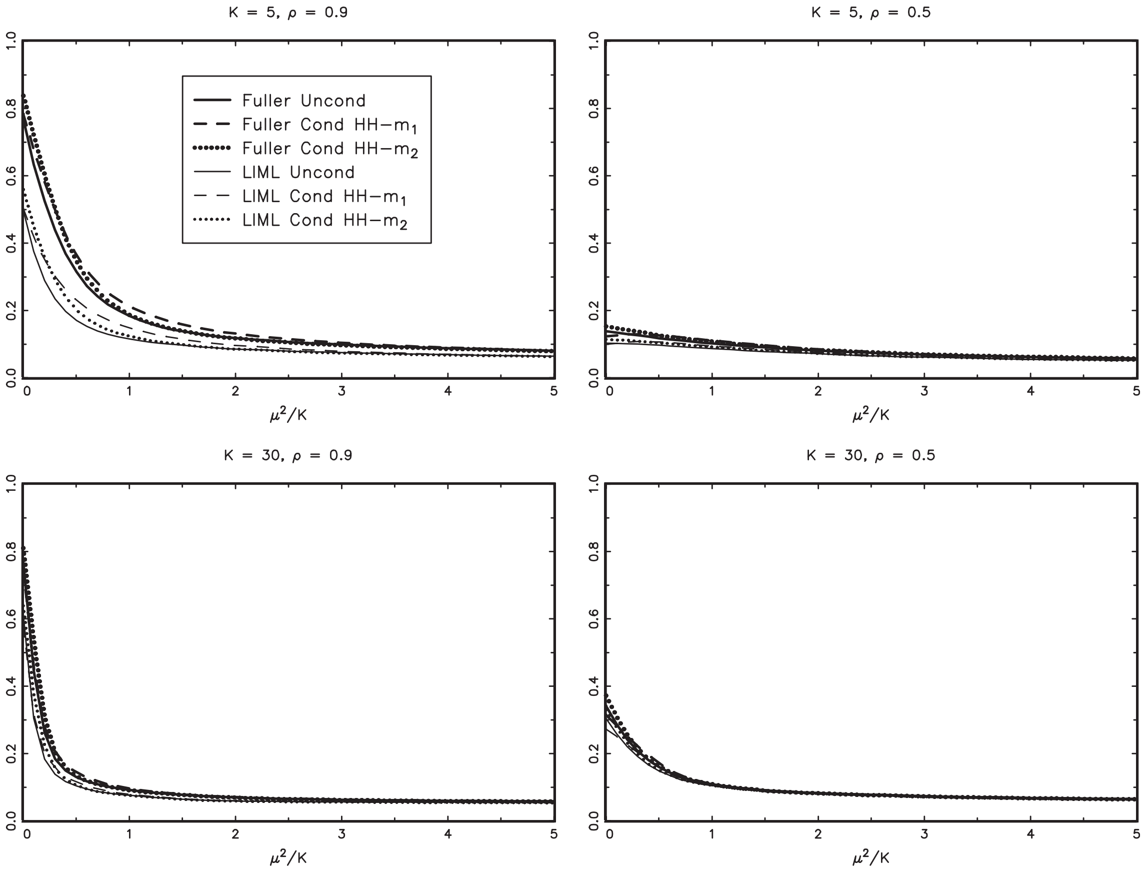

Fig. 4. Rejection rate (size) of a nominal 5% Wald test of $\beta = \beta_0$ based on the Fuller and LIML estimators: unconditional, conditional on $|m_1| \leq 1.645$ ("HH-m_1"), and conditional on $|m_2| \leq 1.645$ ("HH-m_2").

The reason that $\hat{\beta}$ and $\hat{\sigma}_{\boldsymbol{u}}^2$ appear in (3) and (4) is to estimate the incidental parameters β and $\sigma_{\boldsymbol{u}}^2$ that arise in the second-order asymptotics. HH originally proposed estimating the incidental parameters by LIML. The extensive Monte Carlo study of Hahn et al. (2004) suggests, however, that the estimator of Fuller (1977) is a better choice because it is less prone to outliers when instruments are weak.

2.2. Weak instrument asymptotic distribution

The asymptotic representation of m_1 and m_2 is derived under a sequence of alternatives in which the concentration parameter is held fixed as the sample size increases. Following Staiger and Stock (1997), let $\boldsymbol{\Pi} = \mathbf{C}/\sqrt{n}$, where $\mathbf{C}$ is a fixed matrix. Under this nesting, the concentration parameter is

$$\mu^2 = \mathbf{C}' Q_{\mathbf{ZZ}} \mathbf{C} / \sigma_{\nu}^2, \tag{5}$$

where $Q_{\mathbf{ZZ}} = E(\mathbf{Z}'\mathbf{Z}/n)$. If $\mu^2 = 0$, then the instruments are irrelevant and β is unidentified.

Define the 2×2 matrices $\bar{\boldsymbol{\Sigma}}$ and $\mathbf{B}$, where $\bar{\boldsymbol{\Sigma}}_{11}=\bar{\boldsymbol{\Sigma}}_{22}=1$ and $\bar{\boldsymbol{\Sigma}}_{12}=\bar{\boldsymbol{\Sigma}}_{21}=\rho$ and where $\mathbf{B}_{11}=\mu^2$ and $\mathbf{B}_{12}=\mathbf{B}_{21}=\mathbf{B}_{22}=0$. Define $\boldsymbol{\Psi}$ to be a 2×2 random matrix with a noncentral Wishart distribution with K degrees of freedom, covariance matrix $\bar{\boldsymbol{\Sigma}}$, and noncentrality matrix $\mathbf{B}$, and denote the elements of $\boldsymbol{\Psi}$ as

$$\psi=\begin{bmatrix}\boldsymbol{\nu}_1 & \boldsymbol{\nu}_2\\ \boldsymbol{\nu}_2 & \boldsymbol{\nu}_3\end{bmatrix}. \tag{6}$$

Under the conditions of Lemma A1 and Theorem 1 in Staiger and Stock (1997), the following limits hold jointly:

$$\begin{aligned}&(\boldsymbol{y}_1{}'P_{\mathbf{Z}}\boldsymbol{y}_1,\boldsymbol{y}_2{}'P_{\mathbf{Z}}\boldsymbol{y}_1,\boldsymbol{y}_2{}'P_{\mathbf{Z}}\boldsymbol{y}_2)\ \xrightarrow{d}\ (\sigma_{\boldsymbol{u}}^2H_1,\sigma_{\boldsymbol{u}}\sigma_{\boldsymbol{\nu}}H_2,\sigma_{\boldsymbol{\nu}}^2\boldsymbol{\nu}_1),\\ &(\boldsymbol{y}_1{}'M_{\mathbf{Z}}\boldsymbol{y}_1/n,\boldsymbol{y}_2{}'M_{\mathbf{Z}}\boldsymbol{y}_1/n,\boldsymbol{y}_2{}'M_{\mathbf{Z}}\boldsymbol{y}_2/n)\ \xrightarrow{p}\ (\sigma_{\boldsymbol{u}}^2J_1,\sigma_{\boldsymbol{u}}\sigma_{\boldsymbol{\nu}}J_2,\sigma_{\boldsymbol{u}}^2),\\ &\hat{\beta}\ \xrightarrow{d}\ \sigma_{\boldsymbol{u}}\left(\bar{\beta}+\Delta\right)/\sigma_{\boldsymbol{\nu}},\text{ and}\\ &\hat{\sigma}_{\boldsymbol{u}}^2\ \xrightarrow{d}\ \sigma_{\boldsymbol{u}}^2S,\end{aligned} \tag{7}$$

where $\bar{\beta}=\sigma_{\boldsymbol{\nu}}\beta/\sigma_{\boldsymbol{u}}$, $H_1=\bar{\beta}^2\boldsymbol{\nu}_1+2\bar{\beta}\boldsymbol{\nu}_2+\boldsymbol{\nu}_3$, $H_2=\bar{\beta}\boldsymbol{\nu}_1+\boldsymbol{\nu}_2$, $J_1=\bar{\beta}^2+2\rho\bar{\beta}+1$, $J_2=\bar{\beta}+\rho$, $S=1-2\rho\Delta+\Delta^2$, and $\Delta=(\boldsymbol{\nu}_2-\rho\kappa^*)/(\boldsymbol{\nu}_1-\kappa^*)$. If $\hat{\beta}$ is the LIML estimator, then $\kappa^*=\kappa^*_{\text{LIML}}$ =the smallest root of $\det(\boldsymbol{\Psi}-\kappa\bar{\boldsymbol{\Sigma}})=0$. If $\hat{\beta}$ is Fuller's estimator, then $\kappa^*=\kappa^*_{\text{Fuller}}=\kappa^*_{\text{LIML}}-c$, where c is the adjustment constant of Fuller (1977) (see Hahn et al. (2004, Eq. (4)), where the constant is denoted by a).

Substitution of the expressions in the preceding paragraph into (3) and (4) yields

$$m_1\ \xrightarrow{d}\ \frac{|H_1|}{\sqrt{2(K-1)}S|1-(K-1)/\boldsymbol{\nu}_1|}\left[\frac{H_2}{\boldsymbol{\nu}_1}-\frac{H_1}{H_2}+\frac{\Xi^*}{\boldsymbol{\nu}_1H_2}\right]\text{and} \tag{8}$$

$$m_2\ \xrightarrow{d}\ \frac{|\bar{\beta}+\Delta||\boldsymbol{\nu}_1-(K-1)|}{\sqrt{2(K-1)}S}\left[\frac{H_2-(K-2)J_2}{\boldsymbol{\nu}_1-(K-2)}-\frac{H_1-(K-2)J_1}{H_2-(K-2)J_2}\right], \tag{9}$$

where $\Xi^*=(K-1)\{H_1-(K-1)J_1-J_2[2H_2-(K-1)J_2]+J_1\boldsymbol{\nu}_1\}$.

2.2.1. Remarks

1. Both test statistics m_1 and m_2 have $O_p(1)$ limits. This suggests that neither test will reject with probability one asymptotically, regardless of the value of μ^2, and in particular that neither test is consistent against nonidentification.
2. The limiting representations for k-class estimators in Staiger and Stock (1997) are joint with (8) and (9), making it possible to evaluate numerically the asymptotic distribution of k-class estimators conditional on passing the HH pretest.
3. Different k-class estimators have different $O_p(1)$ weak-instrument asymptotic distributions, so the weak-instrument asymptotic distributions of m_1 and m_2 depend on which estimator is used for the incidental parameters.

3. Numerical results

This section examines the asymptotic behavior of the HH pretests by Monte Carlo evaluation of the weak-instrument limits (8) and (9) using 20,000 Monte Carlo draws of the noncentral Wishart random

Table 1
Monte Carlo comparison of finite-sample and weak-instrument asymptotic distributions: m_2 HH test rejection rate and Fuller estimator RMSE

K	μ^2/K	ρ	n	R^2	HH-m_2 rejection rate	RMSE for Fuller ($c=1$)	
						Unconditional	Conditional
5	0.5	0.5	50	0.0476	0.128	0.579	0.578
5	0.5	0.5	100	0.0244	0.135	0.585	0.570
5	0.5	0.5	∞		0.135	0.596	0.592
5	2.0	0.5	50	0.1667	0.098	0.351	0.339
5	2.0	0.5	100	0.0909	0.078	0.365	0.348
5	2.0	0.5	∞		0.089	0.361	0.349
5	0.5	0.9	50	0.0476	0.221	0.542	0.547
5	0.5	0.9	100	0.0244	0.205	0.573	0.577
5	0.5	0.9	∞		0.223	0.574	0.580
5	2.0	0.9	50	0.1667	0.099	0.284	0.283
5	2.0	0.9	100	0.0909	0.088	0.265	0.258
5	2.0	0.9	∞		0.097	0.271	0.261
30	0.5	0.5	100	0.1304	0.065	0.532	0.498
30	0.5	0.5	200	0.0698	0.074	0.496	0.468
30	0.5	0.5	∞		0.079	0.481	0.451
30	2.0	0.5	100	0.3750	0.069	0.174	0.171
30	2.0	0.5	200	0.2308	0.088	0.157	0.153
30	2.0	0.5	∞		0.085	0.156	0.154
30	0.5	0.9	100	0.1304	0.095	0.320	0.304
30	0.5	0.9	200	0.0698	0.094	0.280	0.264
30	0.5	0.9	∞		0.107	0.285	0.265
30	2.0	0.9	100	0.3750	0.079	0.138	0.139
30	2.0	0.9	200	0.2308	0.087	0.138	0.137
30	2.0	0.9	∞		0.092	0.139	0.139

The "HH-m_2 Rejection Rate" is the fraction of times that the m_2-based HH test, calculated using the Fuller ($c=1$) estimator for the incidental parameters, rejects at the 10% significance level (that is, $|m_2|>1.645$). The final two columns report the RMSE of the indicated estimator, either unconditionally (without a pretest) or conditional on passing the HH pretest (that is, if $|m_2|\leq 1.645$). The finite-sample results were computed by Monte Carlo using 1000 draws, using the design described in the text; the results for $n=\infty$ were computed using 20,000 draws from the weak-instrument asymptotic distribution.

variable Ψ. Following HH (footnote 5), we set $\text{var}(\boldsymbol{y}_{1i}|\mathbf{Z}_i)=1, \sigma_{\boldsymbol{\nu}}^2=\text{var}(\boldsymbol{y}_{2i}|\mathbf{Z}_i)=1$, and $\bar{\beta}=-2\rho$ so that $\sigma_{\boldsymbol{u}}^2=1$. With this normalization, the distributions of m_1 and m_2 depend only on K, μ^2, and ρ. Throughout, the estimator of Fuller (1977) with $c=1$ is used to calculate the incidental parameters in m_1 and m_2.

3.1. Power

One definition of weak-instruments is that instruments are weak when the concentration parameter is sufficiently small that conventional first-order asymptotics can be misleading (cf. Stock et al., 2002). Given this definition, the power of the HH test should be high when μ^2/K is small or zero and should equal the size of the test when μ^2/K is large.

The asymptotic power of the two HH tests, at the 10% significance level, is plotted in Fig. 1 as a function of μ^2/K for $K=5$ and 30 and for $\rho=0.9$ and 0.5. For the cases in Fig. 1, the asymptotic power of the 10% HH tests against $\mu^2/K<2$ ranges from 8% to 35%. Generally speaking, the two tests perform similarly. We have considered other values of K, ρ, and $\bar{\beta}$, and the highest rejection rate we found was 35% (we did not conduct an exhaustive search however). For 5% HH tests, the highest rejection rate we found was 27%.

3.2. Post-test estimator performance

HH developed the m statistics to be used as a pretest: if the test fails to reject, then inference should proceed using an estimator that has good second-order properties, for example LIML (HH, p. 179). Thus another way to assess the performance of the statistics is to examine the reliability of post-test inferences, conditional on the HH test failing to reject. For brevity, we focus on post-test inference using Fuller's estimator with $c=1$.

Fig. 2 presents the asymptotic median bias of LIML and Fuller's estimator (a) unconditionally; (b) conditional on passing the m_1 test ($|m_1|\leq 1.645$), and (c) conditional on passing the m_2 test ($|m_2|\leq 1.645$). For Fuller's estimator, there is essentially no difference between the conditional and unconditional median bias curves. For LIML, the median bias is slightly greater conditional on passing a HH test than unconditionally.

Fig. 3 shows the effect of a HH pretest on the absolute estimation error of the two estimators, specifically, the 95% quantile of $|\hat{\beta}-\beta|$ (this measure of estimation error is used instead of the RMSE or MAE because LIML moments need not exist). Both HH pretests reduce the spread of the LIML estimation error, that is, they tend to eliminate the largest outliers. For Fuller's estimator, however, this quantile is not affected by a pretest, and the unconditional quantile for Fuller's estimator is less than or equal to the conditional quantile for the LIML estimator.

Fig. 4 presents the asymptotic null rejection rate (the asymptotic size) of a nominal 5% Wald test of the hypothesis $\beta=\beta_0$ based on LIML and Fuller's estimator (computed using the usual k-class standard error formula), both unconditionally and conditional on passing a HH test. For small values of μ^2/K, the size distortions in both Wald tests can be substantial, especially in the $\rho=0.9$ case, although they tend to be much less than those of the TSLS Wald test (Stock and Yogo, 2005). As in Fig. 2, the conditional and unconditional Wald test size curves are essentially the same.

3.3. Finite-sample results

We performed a Monte Carlo experiment to check whether the weak-instrument asymptotic distributions provide a good approximation to the finite-sample distributions of the HH statistic and post-test estimators. The finite-sample results were computed using 1000 Monte Carlo draws for the system (1) and (2) with i.i.d. normal errors; the parameter settings are the same as described in the first paragraph of this section. To save space, we report a subset of results and focus on the m_2 pretest (with the Fuller ($c=1$) estimator of the nuisance parameter) and the Fuller ($c=1$) post-test estimator.

The results are summarized in Table 1. For a given value of K and μ^2/K, the finite-sample rejection rates of the m_2 statistic (the "HH Rejection Rate" column) are close to each other and to the asymptotic limit for all values of n; by $n=100$, the finite-sample rates generally are within Monte Carlo error of the asymptotic rejection rates. The remaining columns report the RMSE of the Fuller ($c=1$) estimator, first unconditionally then conditional on $|m_2|\leq 1.645$. Again, the finite-sample RMSE is in most cases close to the asymptotic RMSE for $n=50$ (for $K=5$), and in all cases is close for $n=100$.

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