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Measuring money growth when financial markets are changing

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Abstract

This article considers constructing monetary aggregates in the presence of financial market innovations and changes in the relationship between individual assets and output. We propose two procedures for constructing a monetary aggregate with the objective of providing a reliable monetary leading indicator of nominal GDP. In the first, subaggregates discretely switch in and out; in the second, the aggregate's growth is a time-varying weighted average of the growth of the subaggregates, where the weights follow a multivariate random walk. These procedures are used to examine augmenting M2 with stock and/or bond mutual funds. The alternative aggregates are broadly similar to M2, but during 1992-93 they outperform M2.

Key words: Monetary aggregate; M2; Time-varying parameter model; Switching regression; Leading indicators

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1. Introduction

This article addresses the problem of constructing a monetary aggregate when innovations in financial markets change the uses or characteristics of different monetary subaggregates. Our objective is to develop a procedure that automatically adjusts the composition of the monetary aggregate in a way that makes the resulting measure of the money stock a stable leading indicator of nominal GDP and potentially a useful control instrument for altering nominal GDP. Two alternative solutions to this problem are developed and applied to the U.S. experience over the past three decades.

Although we are very much aware that the link between M2 and nominal GDP has been far from perfect, we are nevertheless impressed by the evidence that this link has been sufficiently strong and stable in the past to make the existing M2 series a useful leading indicator of nominal GDP. More specifically, Feldstein and Stock (1994a) confirmed that the rate of change of M2 is a statistically significant predictor of the rate of change of nominal GDP over the period 1959–92. When short-term interest rates are added to the relation, M2 remains statistically significant although interest rates add marginally to the equation's predictive power. Our earlier analysis also showed that the strength and historical stability of this relationship implies that, if the Federal Reserve were able to control M2 optimally, then both the long-term average rate of inflation and the variance of the annual growth rate of nominal GDP could be reduced. Although there is a danger that attempting to use M2 to control GDP would weaken that relation (the 'Goodhart's law' problem), the potential gain appears to be large enough to make further analysis worthwhile.

In practice, the current broad monetary aggregate M2 has been redefined by the Federal Reserve on several occasions. For example, M2 was expanded to include MMMF's and, subsequently, money market deposit accounts. The apparent weakening of the relation between the existing M2 measure and nominal GDP that began in 1992, along with the rise in M2 velocity since 1990 in the face of low and falling interest rates, has caused some analysts to conclude that M2 should be expanded again to incorporate bond and stock funds (see Duca, 1993; Darin and Hetzel, 1994; Orphanides, Reid, and Small, 1994; for discussions of this proposal). Such changes in the definition of the monetary aggregate are potentially appropriate when households and businesses substitute existing or new financial instruments for previous components of the monetary aggregate. Money market deposit accounts and MMMF's were new financial instruments that households could easily substitute for small time deposits. More recently, households have begun to substitute highly liquid bond and stock funds for shorter-maturity money market funds. Since the substitutions are never complete or perfect, the desirability of expanding or redefining the monetary aggregate is an empirical issue.

The decision to redefine the monetary aggregate involves three questions. First, does the proposed new aggregate have a stronger and more stable leading relation with nominal GDP than the existing monetary aggregate? Second, would a decision by the Federal Reserve to shift from the old aggregate to the new alternative weaken public confidence in the Federal Reserve's determination to control the money stock and thereby to limit the rate of inflation? And third, would the new monetary aggregate be more difficult to control than the old one?

The methods described in this paper revise the definition of the monetary aggregate when the rate of growth of the resulting new aggregate would have a stronger relation to the rate of growth of nominal GDP. More specifically, the analysis relates the quarterly rate of growth of nominal GDP to distributed lags of past growth of nominal GDP and the monetary aggregate. The monetary aggregate is changed when the new monetary aggregate improves the ability of that relation to explain changes in nominal GDP.¹

The timing procedures that we discuss are nonjudgmental. Given a list of possible monetary subaggregates (e.g., the existing M1, small denomination time deposits, overnight repurchase agreements, money market deposit accounts, MMMF's, bond funds, etc.), the procedure would automatically decide which ones should be included in 'the' monetary aggregate or how much weight should be assigned to each. This inclusion or weighting decision varies with time. If the list of possible subaggregates is specified in advance, the fact that these changes in the definition of the monetary aggregate are generated by a pre-established rule should reduce suspicion that the redefinition of the aggregate reflects an attempt by the Federal Reserve to avoid a previous commitment to control the growth of the monetary aggregate and thus the future rate of inflation. Indeed, even if the procedure that we develop produced the same redefinitions of the monetary aggregate as the Federal Reserve would choose judgmentally, the ability to arrive at those changes by a prespecified set of rules would be desirable. Alternatively, these procedures can provide a framework for initiating Federal Reserve discussions about changing the monetary aggregate and a standard by which to evaluate judgmental decisions about including or excluding a particular subaggregate.

There is frequently a tradeoff between the controllability of a monetary aggregate and the strength of its link to nominal GDP. Advocates of controlling the monetary base (e.g., McCallum, 1988, 1990) emphasize that it is directly controlled by the Federal Reserve, unlike any of the broader monetary aggregates. Similarly, the Federal Reserve has been much better able to control M1

¹It would be appropriate to examine longer-term relations as an alternative to the quarterly forecasts.

than any broader aggregate because reserve requirements are based on the noncurrency components of M1 while the components of M2 that are not in M1 are no longer subject to reserve requirements. This may explain why the Shadow Open Market Committee and others have focused on the growth of M1. While it is true that the monetary base and M1 are now more controllable by the Federal Reserve than M2, the evidence that we have examined (Feldstein and Stock, 1994a, Sec. 6) indicates that the predictive content of these narrower aggregates is much less than the predictive content of M2. Expanding reserve requirements to all of the bank components of M2 (i.e., M2 excluding MMMF's) would make it possible for the Federal Reserve to control M2 over any period of more than one or two months (Feldstein, 1993).

In designing our current procedures for redefining the monetary aggregates we have not given explicit attention to the controllability of the resulting aggregate. Indeed, one set of experiments in the current paper considers aggregates which include bond and stock mutual funds. Direct control of the size of these funds is well outside the range of current policy options facing the Fed, although no more so than for MMMF's that are now part of M2. This raises the question of how one should interpret the shift by one of our automatic procedures to a monetary aggregate which includes bond and stock funds, since a switch from M2 to M2 plus stock and bond funds is a move away from controllability. One interpretation of such an automatic switch is that this is signalling to the Fed that control of M2 should be deemphasized and replaced with an alternative rule or judgmental procedure, perhaps one based on interest rates. In this sense, the composition of the appropriate aggregate can be interpreted as an indicator of when a monetary aggregate rule may or may not be appropriate. We provide evidence on the magnitude of the improvement in predictability that is achieved by using a less controllable index than M2. However, we do not examine systematically the relative degree of controllability (using interest rates as well as reserves) of M2 and the broader aggregates considered here.

The current paper considers two quite different approaches to redefining the monetary aggregate. The first approach regards the appropriate aggregate as an unweighted sum of certain monetary subaggregates and considers once each quarter whether the set of those subaggregates that is defined to constitute the monetary aggregate should be expanded or reduced in order to improve the predictive link between that overall aggregate monetary stock and nominal GDP. For example, before 1980 the broad M2 money stock did not include MMMF's. The 'switching regressions' procedure presented in Section 2 considers the possibility of adding MMMF's to the narrower pre-1980 aggregate each quarter until the procedure indicates that it is desirable to do so. The results of this approach, presented in Section 3, are consistent with the actual timing of the decision of the Federal Reserve, in the sense that the switching algorithm would have included MMMF's in M2 since the early 1980's. These

results also indicate that expanding the aggregate further to include bond and stock funds would now improve its predictive content, and based on all currently available data these funds switch into the aggregate in 1987.

The second approach models the growth of the monetary aggregate as a weighted sum of the growth of certain monetary subaggregates and reestimates optimal weights each quarter. This ‘time-varying parameter’ procedure, described in Section 4, changes the relative importance of different subaggregates over time and allows new subaggregates to be introduced. Although previous researchers have constructed monetary aggregates with time-varying weights (e.g., Barnett, 1980; Spindt, 1985; Barnett, Fisher, and Serletis, 1992; Rotemberg, Driscoll, and Poterba, 1995; also see Friedman and Schwartz, 1970, Ch. 4.1, for a discussion of unequal weighting schemes), those weights were based on *a priori* theoretical considerations. Despite this virtue, these alternative aggregates have not been widely adopted by practitioners. In contrast, the present paper constructs the aggregate to optimize a prediction function that causes the weights to vary over time. Results are presented in Section 5. Notably, we find empirically that the TVP monetary aggregate is generally similar to the unweighted inclusion–exclusion aggregate derived by the switching regressions method. Despite the differences in the two techniques, the quantitative similarity of the resulting series suggests that the strategy of producing an aggregate to be a stable monetary leading indicator is robust to changes in its implementation.

2. Switching regression model of a monetary indicator

The switching regression model provides a framework for making discrete additions of a new component to an existing monetary indicator. The implementation here assumes that there is a natural order in which to introduce subaggregates into the indicator. For example, in the first empirical application we consider the mutually exclusive subaggregates M1, those subaggregates in M2 excluding M1 and money market mutual funds (MMMF’s), and MMMF’s. Consequently the decisions modeled are when (if ever) to switch from M1 to an aggregate which is M2 excluding MMMF’s, and subsequently when (if ever) to include MMMF’s in M2. This assumption could be relaxed but only at considerable computational cost.²

Let Z_{it} , $i = 1, \dots, I$, denote the level of the i th monetary subaggregate, and suppose that Z_{it} are mutually exclusive. Let S_{it} be the sum of the first through i th

²Each additional switching option increases the number of regressions by approximately a factor of the sample size. As discussed below, the computational cost of increasing the number of subaggregates is much less for the TVP aggregate.

monetary subaggregates, so $S_{it} = \sum_{j=1}^i Z_{jt}$ and let $s_{it} = \Delta \ln S_{it}$. The growth rate of the monetary indicator is defined by

$$m_t(\tau_1, \dots, \tau_{I-1}) = s_{1t} \mathbf{1}(t < \tau_1) + s_{2t} \mathbf{1}(\tau_1 \leq t < \tau_2) + \dots + s_{I-1t} \mathbf{1}(\tau_{I-1} \leq t), \quad (1)$$

where $\mathbf{1}(t < \tau_1)$ takes on a value of one if $t < \tau_1$ and is zero otherwise, etc. Thus the monetary indicator is defined in terms of the growth rates of the increasing family of aggregates S_{it} , where the indicator switches from the $(i - 1)$ th to the i th aggregate at date τ_{i-1} .³

The switching dates are estimated by selecting those dates which produce the aggregate with the greatest ability to forecast GDP growth in a stable forecasting relation. Let $x_t = \Delta \ln GDP_t$. Here, we consider the bivariate forecasting equation

$$x_t = \mu + \alpha(L)x_{t-1} + \gamma(L)m_{t-1}(\tau_1, \dots, \tau_{I-1}) + \varepsilon_t. \quad (2)$$

Thus the forecasting relation between the monetary indicator $m_{t-1}(\tau_1, \dots, \tau_{I-1})$ and GDP growth, conditional on lagged GDP growth, is assumed to be stable. The standard switching regression model has the same set of regressors, with coefficients that change at an unknown date. In contrast, in (2) the coefficients are stable and the only time-varying feature is the monetary aggregate itself.

The switch dates are estimated by least squares. Let $SSR(\tau_1, \dots, \tau_{I-1})$ denote the sum of squared residuals from estimating (2) by OLS, given $\tau_1, \dots, \tau_{I-1}$. The switch date estimator solves

$$\min_{\tau_1 < \dots < \tau_{I-1}} SSR(\tau_1, \dots, \tau_{I-1}). \quad (3)$$

If $\{Z_{it}\}$ are strictly exogenous and the errors are i.i.d. Gaussian, then this procedure would yield the maximum likelihood estimators of the switch dates. More plausibly, $\{Z_{it}\}$ are predetermined but not exogenous and there is no reason to believe the errors to be Gaussian, in which case one might consider other estimators. An alternative would be to choose $\tau_1, \dots, \tau_{I-1}$ to minimize a multi-step-ahead forecast error.

One would of course like to be able to perform statistical inference on the estimated switch dates, in particular testing the hypothesis that there is no switch and constructing confidence intervals for the switch dates in the event

³The level of the aggregate is S_{it} when the growth rate is s_{it} . At τ_{i-1} , the date of switching to the i th aggregate, there is a jump in the level of the aggregate from $S_{i-1, \tau_{i-1}-1}$ to $S_{i, \tau_{i-1}-1}$. This jump does not imply a spike in the reported growth rate as defined in (1), however, because at date τ_{i-1} , the growth rate of $S_{i, \tau_{i-1}-1}$ is used.

that there has been a switch. These questions have been studied in the related change point problem. In the change point model, statistical significance can be studied using maximal Wald or likelihood ratio statistics, or using related tests with explicit break dates; see, for example, Andrews and Ploberger (1994). Also, nonstandard techniques can be used to construct confidence intervals for a break date (Picard, 1985; Bai, 1995). However, the structure of the problem at hand is sufficiently different that these results do not apply directly (here, the coefficients are constant and the series itself is changing, while the reverse is true in the usual change point problem). We hope to be able to provide results about inference in this model at a later date.

3. Empirical results: Switching regression model

The switching regression procedure is examined by performing two experiments. Following Orphanides and Porter (1993), we first examine the switch from M1 to M2 (excluding MMMF's) and the subsequent decision to incorporate MMMF's into M2. The second examines whether M2 might usefully be extended to include bond, or bond and stock, mutual funds. The data are quarterly, 1959:1 to 1993:4. Quarterly money quantities are averages of the monthly dollar values during the quarter. Our growth rate data on MMMF's begin in 1975:3, and stock and bond mutual fund growth rate data begin in 1975:2; these are the first dates at which the instruments are permitted to enter the switching regression aggregate.⁴ All regressions are executed for samples starting in 1960:2, with earlier observations used for initial conditions for lagged variables.

3.1. Results for M1, M2exMMMF, and M2

For this experiment we set $S_1 = M1$, $S_2 = M2$ excluding MMMF's, and $S_3 = M2$. A natural question is what aggregate the switching algorithm would have produced, had it been run in real time. This question is addressed by computing the switching aggregate recursively for subsamples with terminal dates running from 1978 to 1993. The results are summarized in Table 1. For

⁴We thank John Duca for providing the stock and bond fund data. These data are market values and exclude IRA and Keogh accounts and institutional holdings, which are also excluded from M2. By using market values, the data also include capital gains. These two features make these data less like M2. An alternative approach is to use total inflows excluding capital gains; see Collins and Edwards (1994) and Orphanides, Reid, and Small (1994) for discussions of advantages and disadvantages. Different results could of course obtain using other data, and in this sense our results should be taken as illustrative.

Table 1
 Recursively estimated switching dates for switching aggregates

End of sample	Subaggregates: M1, M2exMMMf, M2		Subaggregates: M2, M2 + MFB, M2 + MFB + MFS	
	M1 to MexMMMf	M2exMMMf to M2	M2 to M2 + MFB	M2 + MFB to M2 + MFB + MFS
78:4	71:4	75:3	77:3	78:2
79:4	71:4	75:3	77:3	78:2
80:4	79:1	80:2	76:3	80:3
81:4	79:2	—	77:4	80:3
82:4	71:4	—	77:4	80:3
83:4	71:4	83:1	77:4	80:3
84:4	71:4	83:1	77:4	80:3
85:4	71:4	83:1	77:4	80:3
86:4	60:4	82:4	—	—
87:4	60:4	87:3	87:1	87:2
88:4	71:4	83:1	87:1	87:2
89:4	60:4	87:3	87:1	87:2
90:4	60:4	82:4	87:1	87:2
91:4	60:4	82:4	87:1	87:2
92:4	60:4	—	87:1	87:2
93:1	60:4	92:4	87:1	87:2
93:2	60:4	92:4	87:1	87:2
93:3	60:4	92:4	87:1	87:2
93:4	60:4	—	87:1	87:2

Each row presents results from estimating the switching regression procedure in Section 2 using three lags in the regressions; regressions are run using data from 1960:2 to the date in the first column. For the second and third columns, the three possible aggregates are M1, M2exMMMf, and M2; for the final two columns, the possible aggregates are M2, M2 + MFB, and M2 + MFB + MFS. The entries in the final four columns are the switch date indicated in the column heading, estimated using data through the row entry in the first column. A dashed line indicates that the aggregate did not in fact make the indicated switch during that sample.

example, using data from 1959:1 to 1978:4, the best indicator would have switched from M1 to M2exMMMf's in 1971:4, then would have included MMMf's starting in 1975:3. During 1982, the best indicator would have excluded MMMf's, and subsequently MMMf's reenter only after the recovery from the second early-80's recession is under way. From 1983:2 through 1992:3, the aggregate would have included MMMf's. Although MMMf's are dropped in 1993:4, the objective function is flat: the full-sample estimates $\hat{\tau}_1 = 1960:4$ and $\hat{\tau}_2 > 1993:4$ produce a sum of squared residuals which is only 0.2 % smaller than the estimates for the subsample through 1993:3 of $\hat{\tau}_1 = 1960:4$, $\hat{\tau}_2 = 1992:4$ and is 0.4 % smaller than the estimates for the subsample through 1989:4 of $\hat{\tau}_1 = 1960:4$, $\hat{\tau}_2 = 1987:3$. Thus, with even a small penalty for switching (which

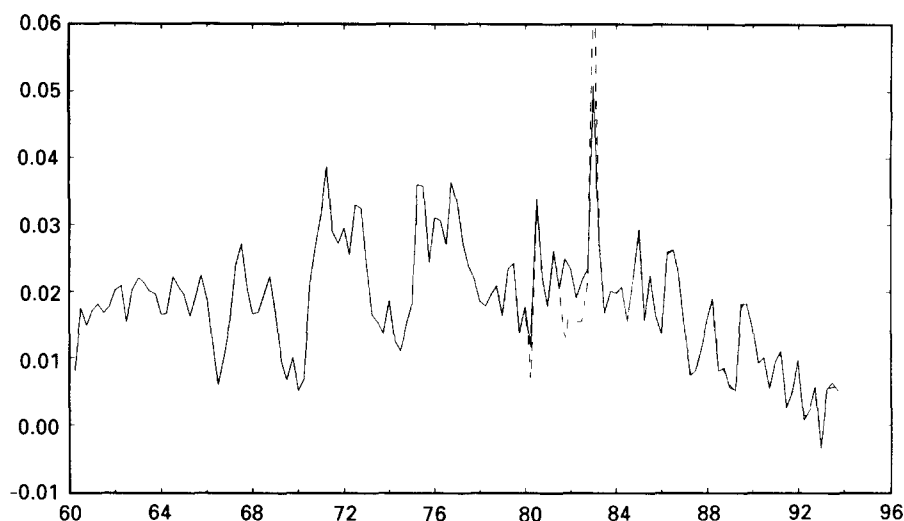


Fig. 1. Quarterly growth of alternative monetary aggregate (dashed line) and M2 (solid line): Simulated real-time series produced by switching regression model.

Recursive estimates for samples from 1960:2–1978:1 through 1960:2–1993:4. Subaggregates: M1, M2exMMMf, M2.

one might plausibly introduce), the real-time aggregate would not have switched and instead would have continued to include MMMF's since 1983:2. Mechanically, MMMF's were dropped in 1993:4 because, during the 1990's, M2exMMMf was growing more rapidly than M2 so the increase in M2exMMMf velocity was not as sharp as for M2 velocity. Thus M2exMMMf was a better predictor than M2 over this episode.

The growth of the monetary aggregate produced by this recursive procedure is plotted in Fig. 1 (the units are quarterly growth in decimals). This aggregate was computed supposing that it is redefined quarterly; for example, the aggregate reported in 1978:4 would have been M2 because the recursive estimate of the break dates would have placed both M2exMMMf's and MMMF's in the aggregate prior to 1978:4, as indicated in the first row of Table 1. Evidently, the simulated real time aggregate is very similar to M2 itself, with the main exception being 1981–1982, when MMMF's would not have been included and when the aggregate would have grown more slowly than did M2.

3.2. Results for bond and stock mutual funds

The second experiment examines whether an alternative aggregate based on M2 and stock and bond mutual funds could be a better economic indicator than

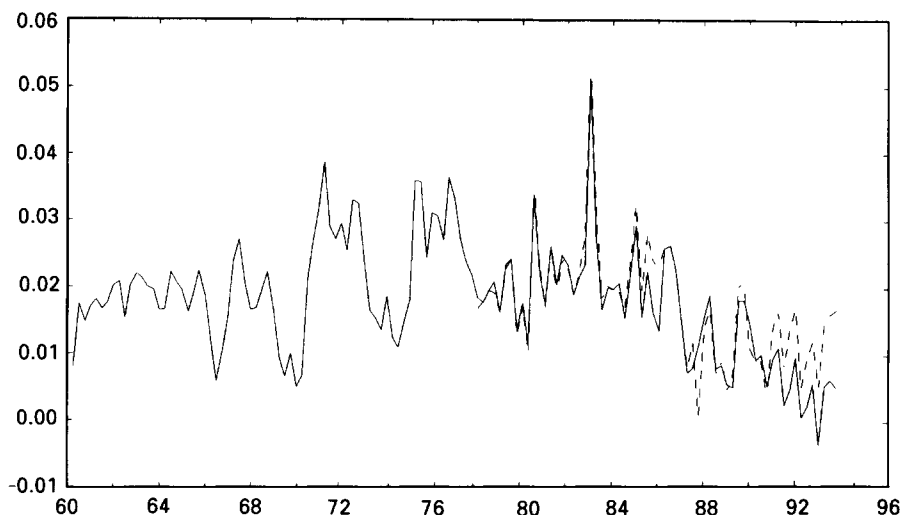


Fig. 2. Quarterly growth of alternative monetary aggregate (dashed line) and M2 (solid line): Simulated real-time series produced by switching regression model.

Recursive estimates for samples from 1960:2–1978:1 through 1960:2–1993:4. Subaggregates: M2, M2 + MFB, M2 + MFB + MFS.

M2 alone. Specifically, we consider results based on the subaggregates M2, bond mutual funds (MFB), and stock mutual funds (MFS), added sequentially, so that $S_1 = M2$, $S_2 = M2 + MFB$, and $S_3 = M2 + MFB + MFS$. The results of the recursive simulation, in which the break dates were estimated over subsamples with increasing terminal dates, are summarized in the final two columns of Table 1, and the simulated real-time switching aggregate is shown in Fig. 2. Through the early 1980's, the simulated real-time series would have included both bond and stock funds, although both stock and bond funds would have been dropped temporarily in 1986. The 1987:4 stock market crash produced a decline in the switching aggregate, but not a sufficiently large forecast error for stock funds to be dropped.⁵ Although the recursive switching aggregate is similar to M2 for most of the sample, from 1991 through 1993 its growth exceeds M2 by approximately 2–3 percentage points at an annual rate.

⁵Results in Feldstein and Stock (1994b) show that if the money data are for the final month of the quarter, rather than the quarterly average used here, MFB and MFS are temporarily dropped after the 1987 stock market crash. Whether MFB and MFS are included immediately after the crash has, however, minor practical significance because, as can be seen from Fig. 2, the growth rates of M2 and M2 + MFS + MFB were similar in 1988 and 1989.

4. Time-varying parameter model of a monetary indicator

The time-varying parameter (TVP) model provides an alternative framework for measuring the rate of change of a monetary aggregate with stable indicator properties. In contrast to the switching model of Section 2, the TVP model produces a series in which the weights on the subaggregates vary over time but in general are neither zero nor one. The growth rate of the aggregate is constructed as a weighted average of the growth rates of the subaggregates. To first order, the growth rate of a conventional aggregate such as M2 can be approximated as a weighted average of the growth rates of its components, where the weights are the time-varying shares of the components in the total aggregate. Weighted averages of growth rates are also used in the monetary services and transaction cost index approaches. For example, Barnett's (1980) Divisia monetary aggregate is constructed in growth rates as a time-varying weighted average of the growth rates of the subaggregates, where the weights depend on the subaggregate's expenditure share based on user costs. Our TVP approach allows the weights on the components to differ from their shares by choosing them to produce an aggregate with a stable predictive relationship to GDP. The use of growth rates here is mainly for computational convenience, since this produces a model which is linear in the (unrestricted) parameters. An advantage of the TVP model over the switching model of the previous section is that there is a well-developed theory of inference in the model, including standard errors on the time-varying weights.

The standard time-varying parameter model proposed by Cooley and Prescott (1973a, 1973b, 1976), Rosenberg (1972, 1973), and Sarris (1973) has time variation in all of the regression coefficients. In contrast, the statistical model considered here presumes a stable relation between the monetary aggregate and output, so only the weights used to construct the aggregate vary over time. The model is

$$\Delta x_t = \alpha_0 + \alpha(L)\Delta x_{t-1} + \gamma(L)(\beta'_t z_{t-1}) + \varepsilon_t, \quad \varepsilon_t \text{ i.i.d. } N(0, \sigma_\varepsilon^2), \quad (4a)$$

where x_t is the logarithm of nominal GDP and z_t is the vector of first differences of logarithms of the various mutually exclusive subaggregates. (By specifying z_t in growth rates, we make the empirically and theoretically plausible assumption that the log-levels of the subaggregates are not cointegrated.) The weights β_t are assumed to evolve as

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \text{ i.i.d. } N(0, \sigma_\eta^2 I), \quad (4b)$$

where ε_t and η_t are independent. The monetary aggregate is $m_t^* = \beta'_{t+1} z_t$. The subaggregates z_t are specified in growth rates. Thus the growth rate of the aggregate is a weighted average of the growth rate of the subaggregates, with weight vector β_{t+1} .

In this model, β_t is an unobserved random variable so the monetary aggregate m_t^* technically is not observable. However, β_t can be estimated using the full data set. Let $\beta_{t|T}$ denote the estimate of β_t given data through time T (that is, given all the available data). Then the estimated monetary aggregate is

$$m_{t|T}^* = \beta'_{t+1|T} z_t. \quad (5)$$

Note that $\beta_{T+1|T} = \beta_{T|T}$, so for the final observation $m_{T|T}^* = \beta'_{T|T} z_T$.

If all the subaggregates exist over the entire time period, then the dimension of z_t is constant. In practice, however, over time new financial instruments become available. This is handled by permitting the dimension of z_t and β_t to expand when a new instrument is introduced, a modification which is conceptually straightforward using the Kalman filter.

The parameters of the model consist of $\theta = (\alpha_0, \alpha(L), \gamma(L), \beta_0, \sigma_\varepsilon^2, \sigma_\eta^2)$. Given θ , $\beta_{t+1|t}$ and $\beta_{t+1|T}$ respectively can be estimated using the Kalman filter and the Kalman smoother for the TVP model. The state vector consists of $(\beta'_t, \beta'_{t-1}, \dots, \beta'_{t-p+1})'$, where p is the order of $\gamma(L)$. The state transition equation is (4b), augmented to handle the lags of β_t . The measurement equation is (4a). This constitutes a standard state space model so the Kalman filter and smoother can be applied directly to yield estimates of β_t and its standard error; see, for example, Harvey (1989). The only remaining issue is the choice of initial conditions for the filter. The convention adopted here depends on the subaggregate. For subaggregates which exist at the beginning of the sample, $\beta_{0|0}$ is set to the share of each subaggregate in the total, so that the first-period growth rate equals the growth rate of an aggregate which is equally weighted in levels. For subaggregates introduced after the beginning of the sample, the initial weight on a new aggregate is set to zero. In both cases the element of the state covariance matrix corresponding to this weight is set to zero in its initial period. The choice of initial conditions has only transitory startup effects, and unreported experiments indicated that our empirical results are insensitive to these initial condition assumptions. Also, for producing the aggregate, the smoothed weights are normalized to add to one in each period; that is, the weights used are $\beta_{i|T}^N / \sum_{i=1}^I \beta_{i|T}$, where $\beta_{i|T}$ are the weights produced by the Kalman smoother.

For our main results, θ is estimated by maximum likelihood. The Kalman filter as just described produces as a byproduct the value of the Gaussian likelihood given θ , and the MLE is obtained by maximizing this likelihood. The likelihood was optimized using a simulated annealing algorithm, modified with a local quadratic search routine.

One part of the analysis, the construction of a simulated real time monetary aggregate, does not use the MLE's. The simulated real-time monetary aggregate is produced by reestimating the parameters every quarter, so that each quarter the data set increases by one observation; the weights on the subaggregates for the final period of the subsample, based on these reestimated parameters, is then used to construct the simulated real-time monetary aggregate. This entails

numerous parameter reestimations, so for this purpose the computationally intensive Kalman filter MLE's are impractical. Instead θ was reestimated by nonlinear generalized least squares (NGLS). In brief, if the regressors are strictly exogenous, the NGLS estimator differs from the MLE only by the approximation $\gamma(L)(\beta'_i z_{t-1}) = \beta'_i(\gamma(L)z_{t-1})$, which was used for numerical convenience. This introduces approximation error into the model, but this error will be small if $\gamma_j^2 \sigma_\eta^2 \text{var}(z_{it}) / \text{var}(\varepsilon_i) \ll 1$ for $j \geq 2$, $i = 1, \dots, I$. This condition is satisfied in the empirical work where this ratio is typically less than 1%. With this assumption and the identity $\beta_i = \beta_0 + \sum_{s=1}^i \eta_s$, the model (4a) and (4b) can be rewritten as

$$\Delta x_t = \alpha_0 + \alpha(L)\Delta x_{t-1} + \gamma(L)(\beta'_0 z_{t-1}) + u_t, \quad (6)$$

where $u_t = (\sum_{s=1}^t \eta_s)'(\gamma(L)z_{t-1}) + \varepsilon_t$. The nonlinearities come through two restrictions: the restrictions placed by $\gamma(L)\beta'_0$ on the coefficients on the lags of z_t , and the restriction that $\gamma(L)$ appears in both the mean and the conditional covariance matrix of (u_1, \dots, u_T) . Were the regressors strictly exogenous, then feasible NGLS would yield the Gaussian MLE. This is readily implemented numerically by (i) obtaining preliminary estimates of $\alpha(L)$, $\gamma(L)$, and β_0 ; (ii) computing the $T \times T$ variance-covariance matrix of (u_1, \dots, u_T) and inverting it; (iii) using this to compute the one-step NGLS estimator, imposing the conditional mean restrictions; and (iv) iterating on this until convergence. This NGLS algorithm was used to produce the subsample estimates in the simulated real-time experiment. Because the regressors are predetermined but not strictly exogenous, this algorithm does not produce the MLE here. However, using the full sample, the point estimates from the NGLS algorithm are similar to the MLE's, as are the implied historical monetary aggregates; this suggests that the simulated real-time results are insensitive to the use of the NGLS estimator rather than the MLE.

5. Empirical results: TVP model

5.1. Results for M1, M2exMMMf and M1, and MMMF's

The first experiment considers an aggregate, the growth of which is a time-varying linear combination of the growth rates of M1, M2R = M2 excluding MMMF and M1, and MMMF. For the initial years after their introduction, the value of MMMF's was small, so small changes in their value resulted in large changes in their growth rates and hence the growth rate of MMMF's has a large variance in these early years. We therefore chose 1980:1 as the first quarter in which MMMF's could enter the monetary aggregate m_t^* ; that is, through 1979:4, we set the weight on MMMF's in m_t^* to zero.

Three sets of MLE's, computed over the full sample, are given in the first three columns of Table 2. In the first two columns σ_η was restricted to 0.025 and 0.05,

Table 2

MLE parameter estimates: TVP model - Estimation period: quarterly, 1960:2-93:4

Notation: $\alpha(L) = \sum_{i=1}^3 \alpha_i L^i$, $\gamma(L) = \sum_{i=1}^3 \gamma_i L^i$

Parameter	Subaggregates: M1, M2R, MMMF			Subaggregates: M2, MFB, MFS		
	(1)	(2)	(3)	(4)	(5)	(6)
α_0	0.0069 (0.0026)	0.0079 (0.0027)	0.0050 (0.0025)	0.0069 (0.0027)	0.0080 (0.0026)	0.0060 (0.0024)
α_1	0.100 (0.091)	0.082 (0.096)	0.133 (0.086)	0.117 (0.090)	0.116 (0.091)	0.086 (0.087)
α_2	0.066 (0.089)	0.068 (0.095)	0.062 (0.087)	0.063 (0.084)	0.077 (0.090)	0.062 (0.088)
α_3	0.033 (0.082)	0.020 (0.084)	0.053 (0.083)	0.016 (0.076)	0.005 (0.068)	0.028 (0.084)
γ_1	0.257 (0.098)	0.218 (0.090)	0.264 (0.123)	0.205 (0.103)	0.178 (0.079)	0.162 (0.129)
γ_2	0.126 (0.135)	0.082 (0.126)	0.178 (0.150)	0.193 (0.122)	0.115 (0.096)	0.257 (0.155)
γ_3	0.030 (0.109)	0.015 (0.103)	0.086 (0.128)	0.010 (0.100)	-0.010 (0.092)	0.100 (0.136)
σ_ϵ	0.00853 (0.00054)	0.00839 (0.00055)	0.00873 (0.00055)	0.00870 (0.00054)	0.00870 (0.00055)	0.00883 (0.00055)
σ_η	0.025*	0.05*	0.00025 (0.00773)	0.025*	0.05*	0.00059 (0.00759)
2 × log-likelihood	1144.836	1143.747	1146.188	1138.711	1135.272	1140.855

M2R = M2 less M1 and MMMF. Parameters were estimated by maximum likelihood using the Kalman filter. The likelihood was maximized by simulated annealing with a minimum of 10,000 evaluations for each optimization.

* Value imposed.

respectively; in the third column, σ_η was estimated along with the other parameters. Notably, the point estimate of σ_η is very small, so that the estimated weights for model (3) vary trivially over the sample. However, the values of the likelihood are relatively close, suggesting that nonzero values of σ_η are also plausible. (Formal distribution theory of the likelihood ratio test statistic when the true value of σ_η is nearly zero appears to be unavailable, so we do not report formal confidence intervals for σ_η .)

We focus on the case $\sigma_\eta = 0.025$. The smoothed estimate of the resulting monetary aggregate, m_{1T}^* , is plotted in Fig. 3a. Fig. 3b presents the smoothed normalized weights, β_{1T}^N ; standard errors on these weights (not plotted), which vary over time and are largest at the end of the sample, typically range from 0.04 to 0.12. Because these weights are on growth rates, they are not immediately comparable to the unit weights on the dollar-valued levels of the various components actually used to construct M2. For a comparison, Fig. 3c plots the effective weights on the growth rates of the components of M2 based on the first-order approximation $\Delta \ln M2_t = \sum_{i=1}^3 w_{it} z_{it}$, where $w_{it} = Z_{it-1} / \sum_{i=1}^3 Z_{it-1}$, i.e., w_{it} is the share of the i th subaggregate.

Overall, the pattern of weights placed on the subaggregates by the TVP model is broadly similar to the pattern of implicit weights they receive in M2. In both aggregates M2R almost always receives the greatest weight, followed by M1 and MMMF's, although M1 receives more weight in the TVP aggregate. During most of the sample the weight placed on MMMF's in the TVP aggregate is less than the weight with which it implicitly enters M2. In fact, the weight placed on MMMF growth is often negative, so that a decline in MMMF's *increases* the growth of this monetary aggregate. However, this result should not be overinterpreted, since the weights on the individual components are imprecisely estimated; for example, the smoothed TVP weight on MMMF's in 1993:4 is -0.04 with a standard error of 0.11. The estimated new aggregate and M2 are notably similar over most of the period, as can be seen from Fig. 3a. The greatest discrepancies between the two are during the 1979–1982 recessions, when the new aggregate grew approximately 6 percentage points (at an annual rate) more slowly than M2, during 1989, and during the final two years of the sample, when it grew somewhat more rapidly than M2. Results for $\sigma_\eta = 0.05$ (not shown) are qualitatively similar, although the weights vary somewhat more than for $\sigma_\eta = 0.025$. Even though the weights differ, the estimated monetary aggregates are similar for both values of σ_η .

The TVP model presumes that the relation between the monetary aggregate and output is stable. This overidentifying restriction can be tested. To check for instability in the presumed time-invariant parameters α_0 , $\alpha(L)$, $\gamma(L)$, and σ_ε^2 , we reestimated the model by maximum likelihood (imposing $\sigma_\eta = 0.025$) on the first and second halves of the data set, specifically over 1960:2–1976:4 and over 1977:1–1993:4. The resulting likelihood ratio statistic, which has a χ_8^2 distribution, is 4.76 which has a p -value of 0.78. This provides no evidence against the

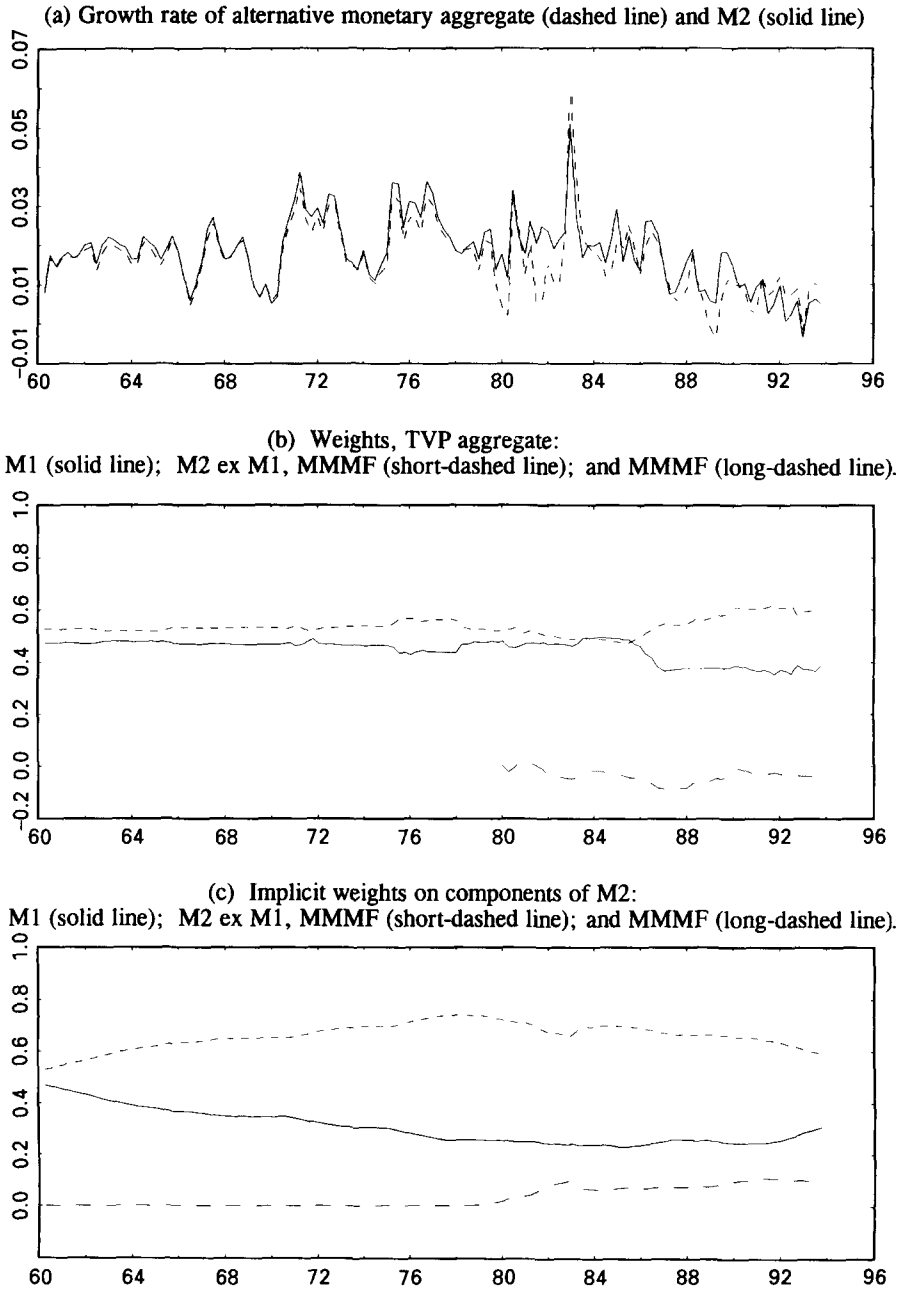


Fig. 3. Growth of TVP aggregate ($\sigma_n = 0.025$) and M2 and weights on their subaggregates.

Subaggregates: M1, M2exM1, MMMF, and MMMF (80:1 start).

overidentifying restriction that the time variation enters only through the weights β_t .⁶

5.2. Introduction of stock and bond mutual funds

The three mutually exclusive subaggregates considered in this experiment are M2, bond mutual funds (MFB), and stock mutual funds (MFS). Estimates of the parameters of the model are given in the final three columns of Table 2. Like MMMF's, stock and bond mutual funds had relatively small dollar values in the 1970's and consequently had highly variable growth rates. We therefore use 1981:1 as the quarter in which MFB and MFS are first considered for inclusion in the new aggregate, that is, the weights on MFB and MFS are set to zero through 1980:4.

Figs. 4a and 4b show the TVP aggregate and corresponding smoothed normalized weights for $\sigma_\eta = 0.025$; typical standard errors for the weights are 0.12 for M2, 0.05 for MFB, and 0.06 for MFS, although these vary over time. Fig. 4c shows the implicit weights on growth rates of the subaggregates for the total, M2 + MFB + MFS, that is, the shares. A noteworthy feature of the results is the different weights placed on MFB and MFS. From 1982:3 on, bond funds receive negative weight in the new aggregate, although the weights are typically less than their standard errors. In contrast, stock funds receive positive weights which are similar to the implicit weights they receive in the M2 + MFB + MFS aggregate. Because the growth rate of the stock funds reflects both changes in stock prices and net flows into the funds, without further analysis we cannot say whether it is the flow of funds or changes in stock prices which are driving this positive weight on stock funds. It is not surprising, however, that including a subaggregate which is determined in part by stock prices improves the predictive content of M2.

The TVP monetary aggregate including stock and bond funds has a growth rate similar to M2, although their growth rates differ in some episodes. For example, the growth of the aggregate including stock funds decreased with the stock market crash of 1987. Over 1993, the new aggregate exhibited a slightly higher average growth rate than M2.

Simulated real time aggregates, estimated by NGLS, are presented in Fig. 5 for selected terminal dates. For an additional comparison with Fig. 4, the results

⁶We also considered how this procedure would have worked in a simulated real-time setting, in which the parameters as well as the weights are reestimated in each period. For this exercise, $\alpha(L)$ and $\gamma(L)$ were estimated by NGLS for the recursive subsamples. In general the subsample and full-sample TVP aggregates are similar. For example, all subsamples yield TVP aggregates which have slower growth than M2 during the 1979–82 period (for details see Feldstein and Stock, 1994b).

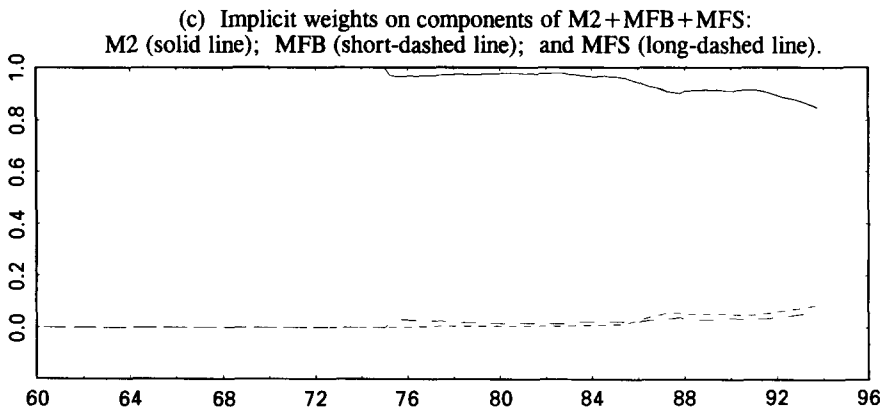
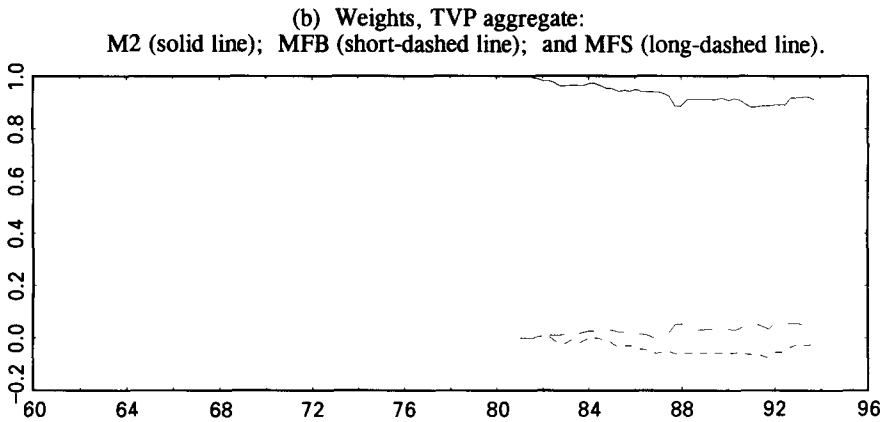
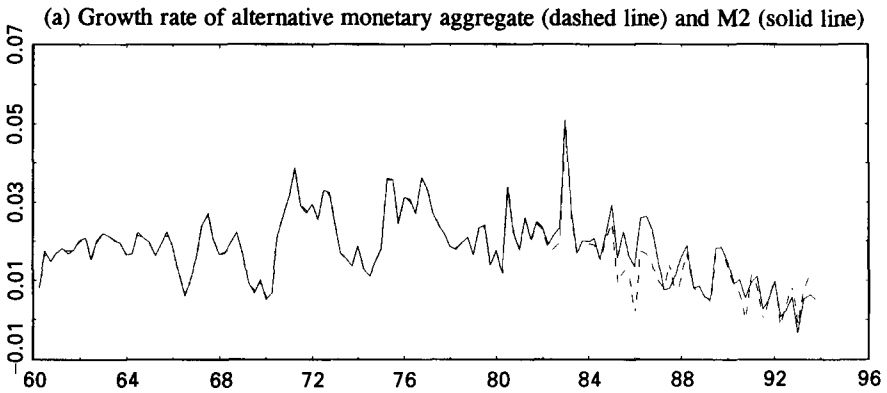


Fig. 4. Growth of TVP aggregate ($\sigma_\eta = 0.025$) and M2 and weights on their subaggregates.
Subaggregates: M2, MFB (81:1 start), MFS (81:1 start).

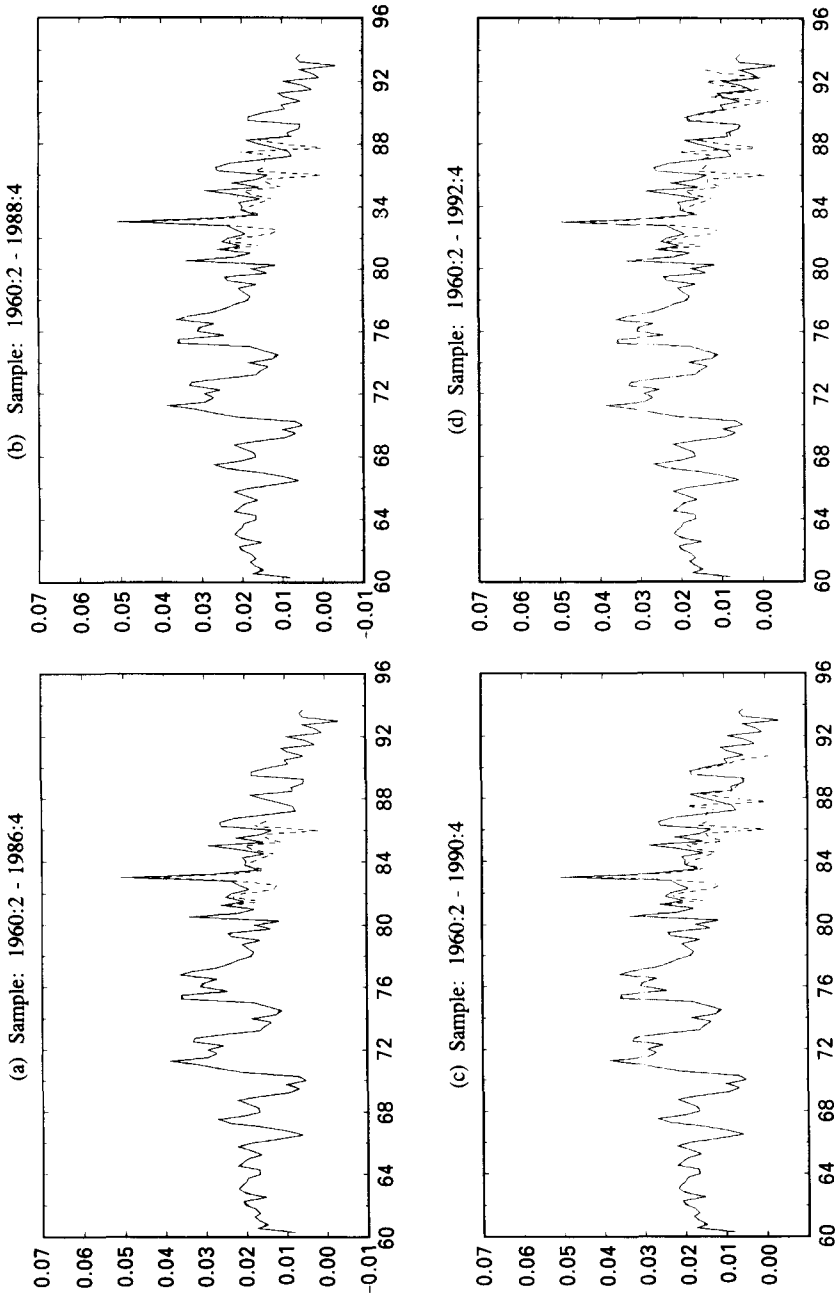


Fig. 5. Growth rate of recursive TVP aggregate (dashed line) and M2 (solid line).

Subaggregates: M2, MFB, MFS; $\sigma_v = 0.05$, 3 lags.

in Fig. 5 are for $\sigma_\eta = 0.05$. In general the recursive TVP aggregates are similar to M2 through 1990, with some brief exceptions; the two exhibit a slight persistent divergence starting in late 1991. In addition, the recursive aggregates in Fig. 5 for $\sigma_\eta = 0.05$ are similar to the full-sample aggregate for $\sigma_\eta = 0.025$ in Fig. 4. As seen in Fig. 5d, by the end of 1992 this aggregate had diverged sufficiently to suggest that it might be appropriate to switch or to consider tracking this broader TVP aggregate as a monetary indicator.

6. Recent performance of the alternative aggregates

The early 1990's saw an historically unprecedented deterioration of the relationship between M2 and nominal GDP, most notably an increase in M2 velocity in the face of low and declining interest rates. One way to judge the importance of using these alternative aggregates is therefore to compare the forecasts of GDP growth based on the switching and TVP aggregates which consider stock and bond mutual funds, to forecasts based on M2. One-quarter-ahead forecasts using the various monetary aggregates over the period 1988:1–1993:4 are plotted in Fig. 6, and root mean squared errors (RMSE's) are summarized in Table 3. The forecasts are computed recursively to simulate real-time implementation of the alternative aggregates. For example, the forecast of GDP growth in 1991:4 based on M2 is computed from the regression of GDP growth on its lags and on lagged values of M2 growth over the period 1960:2–1991:3 (earlier values are used for initial conditions). The switching forecasts are based on the simulated real-time switching aggregate plotted in Fig. 2, that is, the switching aggregate and its GDP forecasting equation are computed using data through the quarter prior to the quarter being forecasted. The TVP aggregate is based on the weights for $\sigma_\eta = 0.025$. As discussed previously, it is computationally unrealistic to compute MLE's of the time-invariant parameters of the TVP model recursively, so the TVP forecast is based on the full-sample parameter estimates (Table 2, model (4)), but the weights used are the Kalman filter estimates $\beta_{i|t}^N$, where t denotes the quarter being forecasted, i.e., not the smoothed weights.

The RMSE's in Table 3 suggest that the choice of monetary aggregate matters over this period. For the entire period 1988–1993, the recursive switching aggregate has the smallest RMSE, followed by the TVP aggregate and M2. Although none of the monetary aggregates forecasted the 1990 recession, especially the nearly zero growth in the fourth quarter of 1990, both the switching and TVP aggregates forecasted growth over the final two years more accurately than M2. Because these are recursive forecasts, this is not an artifact of the aggregate being defined *ex post* as that which forecasts GDP well over this period. Rather, this can be taken as reflecting a deterioration in the forecasting performance of M2 relative to the broader aggregates.

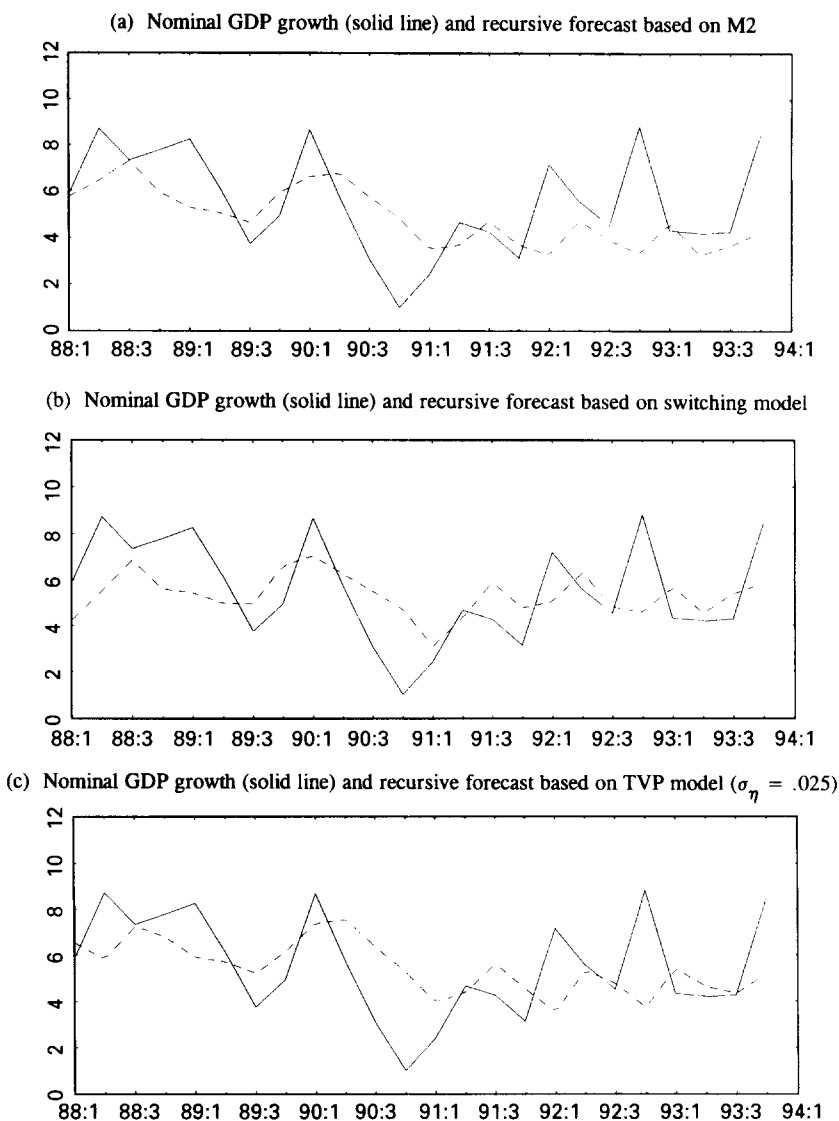


Fig. 6. Actual quarterly nominal GDP growth (at an annual rate) and forecasts based on monetary aggregates.

While these results are promising, they are limited in several regards. The analysis has focused entirely on the bivariate M2–output relation. One extension is to higher-dimensional models which include interest rates. Of particular interest are models in which the weights are determined in part so that the

Table 3

RMSE's of recursive forecasts of quarterly GDP growth using various monetary aggregates (percent growth over the previous quarter at an annual rate) – Subaggregates: M2, MFB, MFS

Period	Recursive forecast based on:		
	M2	Switching M2*	TVP M2*
1988:1–89:4	1.60	1.99	1.54
1990:1–91:4	1.93	1.88	2.24
1992:1–93:4	2.86	2.03	2.52
1988:1–93:4	2.19	1.97	2.14

Entries are RMSE's of one-quarter-ahead forecasts computed recursively, that is, using data through the quarter preceding the forecast date, with the exception that the TVP model uses recursive weights but full-sample estimates for the time-invariant parameters. For the TVP model, σ_η was set to 0.025. All regressions included a constant, three lags of GDP growth, and three lags of the relevant monetary aggregate as discussed in the text.

velocity and an interest rate are cointegrated. This would entail extending the foregoing bivariate regressions to error correction specifications.

7. Concluding comment

This paper has developed two alternative statistical procedures for producing a monetary aggregate which is more highly correlated with future values of nominal GDP. The first method changes the set of monetary subaggregates that is included in the monetary aggregate at any time, while the second changes the weights on the growth rates of the subaggregates.

It is not clear whether the recent shifts of funds from traditional M2 components (including money market mutual funds) into stock and bond funds represents a one-time portfolio reallocation or the start of a period in which funds will shift back and forth among these asset classes in response to yield differences. If this is a one-time portfolio reallocation, the velocity of the traditional M2 aggregate should return to the same degree of stability that prevailed in the past. Although there was substantial volatility in this velocity, Feldstein and Stock (1994a) showed that better controlling M2 had the potential for reducing the volatility of nominal GDP relative to the historical experience. If however the behavior of asset holders has changed so that there are substantial and unpredictable shifts of funds between the old M2 and the bond and stock mutual funds, the Feldstein–Stock results may no longer be applicable and it may be appropriate for the Federal Reserve to shift its attention to the expanded M2; only time will tell.

The ability to create alternative monetary aggregates that are more highly correlated with future nominal GDP than is current M2 highlights the importance of the tradeoff between the controllability of a monetary instrument and the strength of that instrument's relation to nominal GDP. When all of the components of M2 were bank liabilities, the Federal Reserve could in principle control M2 quite precisely by changing the supply of reserves through open market operations. In practice, however, reserves are required for only some of the components of M2, and different subaggregates have different reserve ratios. This reserve requirement policy precludes a precise link between open market operations and M2. The Federal Reserve nevertheless controlled M2 indirectly albeit less precisely through its open market operations.

The Fed's decision to expand M2 in 1980 to include money market mutual funds issued by nonbank financial institutions sacrificed even the theoretical ability to control M2 in a precise way by open market operations since nonbank institutions could not be subject to reserve requirements. As a practical matter, however, the change in controllability was only a matter of degree since the Federal Reserve had not previously imposed reserve requirements on all components of M2. After 1980 the Federal Reserve therefore continued to control the newly defined and broader M2 in much the same way that it had before: by changes in reserves and the resulting changes in interest rates that affected the behavior of banks and asset holders. Switching attention to an expanded aggregate that adds bond and stock funds to M2 would presumably (although not necessarily) make the aggregate more difficult to control than the existing M2. Since the principle of a monetary aggregate that includes nonbank liabilities has already been accepted, expanding M2 to include other types of mutual funds would in practice be a matter of degree rather than a fundamental change in controllability.

How then should results from the proposed procedures be used? Our aim is not simply to find a better predictor of nominal GDP, which would lead to using other variables in addition to monetary aggregates. Rather, the purpose of revising the monetary aggregate is to provide information about the state of monetary policy and perhaps to provide a better instrument for controlling nominal GDP.

At a minimum, evidence that an alternative aggregate is more closely linked to future nominal GDP than M2 can serve as a warning against placing undue reliance on the existing M2 aggregate. For example, the slow growth of M2 that began in 1992 should be seen as a less worrying development to the extent that M2 is no longer as appropriate a monetary aggregate as an expanded measure that grew more rapidly during these recent years. If the Federal Reserve is unable to control the broader aggregate, it may be appropriate to shift to an alternative instrument like the federal funds rate.

More broadly, an observed divergence of the alternative monetary aggregates from M2 (or, more generally, some other reference aggregate), combined with

the lack of full controllability of M2, raises several policy options: (1) return to a narrower measure which excludes money market mutual funds and increase controllability of that aggregate by imposing uniform reserve requirements; (2) continue to make policy in terms of the existing M2 aggregate, using open market operations to guide the evolution of M2 even though control is far from complete; (3) shift attention to an expanded aggregate that includes (at least for now) the bond and stock funds, using open market operations to guide it even though controllability may be less than with the existing M2 measure; (4) abandon an aggregates-based strategy and use the federal funds rate or some more complex measure of monetary conditions as a guide to federal reserve policy; and (5) abandon all attempts to measure the link between monetary policy and future GDP or other economic conditions and just change the federal funds rate or the stock of reserves by 'small' amounts in response to the gap between the forecast and target levels of nominal GDP or some other measure of economic performance.

These alternatives suggest four subjects for further research. First, would a return to a narrower but completely controlled M2 improve the link between Federal Reserve policy and nominal GDP? That is, would the improved controllability of the narrower M2 more than offset the weakened correlation with GDP that results from omitting money market mutual funds from the M2 measure? Second, how much controllability would in fact be lost by going from the current M2 to the expanded M2 that includes stock and bond funds? Would it be better to use the more controllable current M2 than the less controllable expanded M2 that is more highly correlated with future changes in nominal GDP? Third, can a useful rule be developed, based on observed volatility or forecast errors, that indicates when a policy of controlling some monetary aggregate should be abandoned in favor of using an interest rate measure of monetary policy? Fourth, when does experience imply that the ability of the Federal Reserve to assess the link between any measure of monetary policy (either aggregates or interest rates) and economic performance is so limited that it should abandon all such relations and rely on a policy of small changes, despite the fact that lags make it difficult to know the effects of such policies for a considerable period of time?

References

- Andrews, D.W.K. and W. Ploberger, 1994, Optimal tests when a nuisance parameter is present only under the alternative, *Econometrica* 62, 1383–1414.
- Bai, J., 1995, Least absolute deviation estimation of a shift, *Econometric Theory* 11, 403–436.
- Barnett, W., 1980, Economic monetary aggregates: An application of index number and aggregation theory, *Journal of Econometrics* 14, 11–48.
- Barnett, W., Douglas Fisher, and Apostolos Serletis, 1992, Consumer theory and the demand for money, *Journal of Economic Literature* 30, 2086–2119.

- Collins, S. and C.L. Edwards, 1994, An alternative monetary aggregate: M2 plus household holdings of bond and equity mutual funds, *Federal Reserve Bank of St. Louis Review* 76, no. 6, 7–29.
- Cooley, T.F. and E.C. Prescott, 1973a, An adaptive regression model, *International Economic Review* 14, 364–371.
- Cooley, T.F. and E.C. Prescott, 1973b, Tests of an adaptive regression model, *Review of Economics and Statistics* 55, 248–256.
- Cooley, T.F. and E.C. Prescott, 1976, Estimation in the presence of stochastic parameter variation, *Econometrica* 44, 167–184.
- Darin, R. and R. Hetzel, 1994, A shift-adjusted M2 indicator for monetary policy, *Federal Reserve Bank of Richmond Economic Quarterly* 80, no. 3, 25–47.
- Duca, John V., 1993, Should bond funds be included in M2?, *Journal of Banking and Finance*, forthcoming.
- Feldstein, M., 1993, The recent failure of U.S. monetary policy (The 1992 Tinbergen lecture of the Netherlands Royal Economic Society), *De Economist* 141, no. 1, 29–42.
- Feldstein, M. and J.H. Stock, 1994a, The use of a monetary aggregate to target nominal GDP, in: N.G. Mankiw, ed., *Monetary policy* (University of Chicago Press for the NBER, Chicago, IL) 7–62.
- Feldstein, M. and J.H. Stock, 1994b, Measuring money growth when financial markets are changing, NBER working paper no. 4888.
- Friedman, M. and A. Schwartz, 1970, *Monetary statistics of the United States* (Columbia University Press for the NBER, New York, NY).
- Harvey, A.C., 1989, *Forecasting, structural time series models, and the Kalman filter* (Cambridge University Press, Cambridge).
- McCallum, B.T., 1988, Robustness properties of a rule for monetary policy, *Carnegie-Rochester Conference Series on Public Policy* 29, 175–203.
- McCallum, B.T., 1990, Targets, indicators, and instruments of monetary policy, in: W.S. Haraf and P. Cagan, eds., *Monetary policy for a changing financial environment* (AEI Press, Washington, DC) 44–70.
- Orphanides, A. and R. Porter, 1993, Alternative methods of monetary aggregation, Manuscript, May (Board of Governors of the Federal Reserve System, Washington, DC).
- Orphanides, A., B. Reid, and D.H. Small, 1994, The empirical properties of a monetary aggregate that adds bond and stock funds to M2, *Federal Reserve Bank of St. Louis Review*, Nov./Dec., 31–51.
- Picard, D., 1985, Testing and estimating change-points in time series. *Advances in Applied Probability* 176, 841–867.
- Rosenberg, B., 1972, The estimation of stationary stochastic regression parameters re-examined, *Journal of the American Statistical Association* 67, 650–654.
- Rosenberg, B., 1973, The analysis of a cross-section of time series by stochastically convergent parameter regression, *Annals of Economic and Social Measurement* 2, 461–484.
- Rotemberg, J., J. Driscoll, and J. Poterba, 1995, Money, output, and prices: Evidence from a new monetary aggregate, *Journal of Business and Economic Statistics* 13, 67–83.
- Sarris, A.H., 1973, A Bayesian approach to estimation of time varying regression coefficients, *Annals of Economic and Social Measurement* 2, 501–523.
- Spindt, P., 1985, Money is what money does: Monetary aggregation and the equation of exchange, *Journal of Political Economy* 93, 175–204.