CHAPTER 15

Hysteresis and the evolution of postwar U.S. and U.K. unemployment

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Abstract: Hysteresis in unemployment can be characterized as a dependence of the persistence in unemployment on the level of, or on changes in, unemployment. This chapter presents an empirical investigation of this possible nonlinear behavior, in which the persistence and conditional heteroscedasticity of unemployment are allowed to depend on its recent history. Both U.S. and U.K. postwar unemployment are found to exhibit substantial nonlinearities of this form, with high and increasing unemployment corresponding to decreased persistence in both countries.

1 Introduction

The experience of the United States and Europe during the 1980s has renewed interest in the evident persistence of high levels of unemployment. Time-series characterizations of this phenomenon take two forms. First, a statistical interpretation of this persistence, given by Blanchard and Summers (1986a), is that the best forecast of unemployment in a given quarter is very nearly the unemployment rate in the previous quarter; that is, unemployment appears to have an autoregressive root nearly equal to one. Second, the notion that some economies can become "stuck" at high levels of unemployment suggests that the serial correlation of the unemployment rate might itself depend on the level of unemployment, with greater correlation occurring at high rather than at low levels. This second characterization is one interpretation of the proposition that unemployment might exhibit "hysteresis" as discussed by Tobin (1980) and Blanchard and Summers (1986a, b, c), in the sense that continuing high unemployment can be associated with approximately constant inflation rates.

The author thanks R. Cooper, J. Geweke, N. Gottfries, S. R. G. Jones, and T. Sargent for helpful discussions and suggestions. This chapter was written while the author was visiting the Institute for International Economic Studies at the University of Stockholm. This research was supported in part by NSF grant #SES-84-08797.

Taken together, these two observations suggest that unemployment might exhibit substantial dependence at high levels - close to the dependence of a random walk, with a relatively small innovation variance - but might exhibit less dependence (and a greater innovation variance) at low or at rapidly changing levels. This state-dependent serial correlation can be thought of as a potential "nonlinear" feature of the unemployment series in the sense that it would not be present were unemployment truly generated by a stationary linear time-series model. The idea that unemployment might display important nonlinearities has been investigated before. For example, Neftci (1984) and DeLong and Summers (1986) provide empirical evidence that the evolution of U.S. unemployment is asymmetric over cyclical expansions and contractions; Brock and Sayers (1986) also find evidence of nonlinear structure in the unemployment rate. However, these authors investigate these nonlinearities using statistical techniques that shed little light on the hysteresis hypothesis that the conditional heteroscedasticity or correlation in unemployment depends on its level or its rate of change.

The purpose of this chapter is to examine empirically the persistence and hysteresis of aggregate unemployment. *Persistence* is interpreted as meaning that the largest autoregressive root of the unemployment process is near one, and *hysteresis* is interpreted as state- or time-dependent conditional correlations and heteroscedasticity. One way to look for this latter feature is to apply an existing test for nonlinearities in time-series data. Unfortunately, this approach can yield information about whether nonlinearities are present but rarely elucidates their form. Instead, the strategy adopted here is to study nonlinear patterns in unemployment using a parametric nonlinear time-series model that is unconditionally stationary and mean-reverting but has serial correlation and heteroscedasticity that can depend on the recent history of the series.

The nonlinear time-series model considered here is motivated both by the desire to link conditional dependence with conditional heteroscedasticity and by a simple argument based on flows into and out of unemployment. Specifically, in a precise sense, unemployment evolves more rapidly at higher than at lower flow rates. Thus the unemployment rate can be thought of as having an "operational" time scale that is a nonlinear transformation of the calendar time scale on which it is observed, where the transformation depends on unemployment flow rates. When observed at regular intervals in calendar time, at high flow rates the innovations to unemployment appear large, and the usefulness of lagged unemployment in predicting future unemployment is diminished, relative to periods of low flow rates.

This argument, formalized in Section 2, suggests using a time-deformation model (Stock, 1987, 1988) to analyze the nonlinearities in the unemployment rate. In this model, unemployment is assumed to evolve according to a linear, time-invariant, stochastic differential equation on a (continuous) "economic" or operational time scale. This time scale is nonlinearly related to the calendar time scale on which unemployment is observed, with the time transformation depending on exogenous variables or lagged values of unemployment itself. Qualitatively, the state-dependent autoregressive coefficients inherent in the time-deformation model are capable of capturing the concept of hysteresis informally used to describe the recent European experience.

The empirical analysis begins in Section 3 with an examination of measures of persistence in U.S. and U.K. unemployment in linear, discrete-time, autoregressive models. As has been widely noted, both series have a root near one, with U.K. unemployment exhibiting slightly greater average persistence than the U.S. series.

Estimated time-deformation models are presented in Section 4. The empirical strategy is to estimate several models with different variables determining the transformation between operational and calendar time. The statistical significance of these variables and the estimated transformations themselves summarize the nonlinear patterns being captured by the time-deformation model. Although the theoretical discussion in Section 2 is used to guide the choice of variables to enter the time-scale transformation, the objective of the empirical work is not to test the formal theory but rather to use the theory as a starting point for quantifying nonlinear patterns in unemployment. This investigation reveals statistically significant time-scale nonlinearities that differ in important ways between the United States and the United Kingdom. In both countries, the serial correlation in unemployment appears to be *less* at high levels than at low levels of unemployment. Conclusions from this analysis are summarized in Section 5.

2 Unemployment flows and time deformation

This section provides a theoretical motivation for modeling unemployment as obeying a linear, continuous-time stochastic differential equation in operational time, where operational and calendar time are related by some nonlinear transformation. The development starts with the identity

$$\frac{du}{dt} = \alpha(1-u) - \beta u, \tag{1}$$

where u is the unemployment rate, du/dt is its time derivative, and α and β are, respectively, the instantaneous flow rates into and out of unemployment.

In the context of search theory, if there is no labor-force participation decision, then β in (1) can be interpreted as the probability that an unemployed worker finds a job at a time t multiplied by the fraction of jobs that the worker finds acceptable, given a reservation wage and an assessment of the current distribution of wage offers; see, for example, Diamond (1982) or Pissarides (1985). The flow rate into unemployment α can similarly be interpreted as the probability of a layoff or quit. Theoretical and empirical work in search theory examines the behavior of unemployment under various assumptions about α and β. For example, Pissarides (1985) models β as depending on the reservation wage of workers and on the availability of offers as captured by the ratio of vacancies to unemployment; Bjorklund and Holmlund (1981) consider as well the possibility that price misperceptions result in incorrect judgments concerning the real offer distribution.

Informally, the flow rates α and β determine the speed of evolution of the unemployment process: Unemployment evolves more rapidly at high than at low flow rates. To make this precise, I adopt a particular stochastic specification of α and β. Let $\rho(t)$ be a positive deterministic function of time (this will be relaxed later), and let $\epsilon(t)$ be a mean-zero unforecastable continuous-time innovation with unit variance. For tractability, suppose that the predetermined components of α and β are proportional, so that increased flows into unemployment are associated with increased flows into jobs. In addition, suppose that α and β have a common stochastic component such that an unpredictable increase in α is exactly offset by the same unpredictable decrease in β and that this unpredictable component is proportional to $\rho(t)$. Thus,

$$\alpha = \alpha_0 \rho(t) + \sigma\epsilon(t)\rho(t), \tag{2a}$$

$$\beta = \beta_0 \rho(t) - \sigma\epsilon(t)\rho(t), \tag{2b}$$

where σ, α_0, and β_0 are positive parameters.

Assumption (2) captures some important features of the flow of workers into and out of unemployment. If movements into unemployment arise because workers hope to find better jobs, it is plausible that the predictable parts of the flow rates into and out of unemployment are closely related; this is captured by including the term $\rho(t)$ in the two expressions. In addition, an innovation that induces workers to reduce their quit rate seems likely also to induce the unemployed to accept more jobs, suggesting the opposite signs on the innovation terms in α and β. Finally, the assumption of multiplicative rather than additive errors is consistent with empirical emphases on log-linear specifications. To the extent that workers become unemployed because of layoffs, however, assumption (2) is more problematic. On the one hand, it seems likely that many of the variables entering the decision of unemployed workers to accept an offer also affect the layoff decisions of firms. For example, forecasts of future prices enter into assessments of both the future product demand (and thus of the need for layoffs) and the real value of a nominal wage offer. On the other hand, there is no economic reason that these factors should enter α and β through the single function $\rho(t)$ as presumed in (2). Moreover, (2) assumes that a positive innovation in the layoff rate is associated with an equal decrease in the hiring rate; because layoffs are typically costly, the firm might prefer – in response to an innovation – first to reduce hiring or to lay off by attrition, and only second to lay off current employees directly.[1] The point is not, however, to develop a fully specified model of unemployment flows. Rather, it it merely to motivate a simple nonlinear model of the unemployment rate in which the transition probabilities, and in particular $\rho(t)$, have the interpretation as the rates of change of the operational time scale of unemployment relative to its calendar time scale.

Using the specifications (2) for the flow rates, one can write the unemployment rate in terms of $\rho(t)$ and $\epsilon(t)$ by substituting (2) into (1):

$$\rho(t)^{-1}\frac{du}{dt}=\alpha_0-\theta u(t)+\sigma\epsilon(t), \tag{3}$$

where $\theta=\alpha_0+\beta_0$. Because α_0 and β_0 in (2) are positive, it follows that θ is positive, so (3) is stable (a condition assumed henceforth). Note that the differential equation (3) satisfies the integral equation

$$u(t)=\int_{-\infty}^{g(t)}[\alpha_0+\sigma\tilde{\epsilon}(r)]e^{-\theta[g(t)-r]}\,dr, \tag{4}$$

where $g(t)=\int_{-\infty}^{t}\rho(\tau)\,d\tau$ and $\tilde{\epsilon}(s)=\epsilon(g^{-1}(s))$, where $g^{-1}(s)$ exists because $\rho(t)$ is positive, so $\epsilon(t)=\tilde{\epsilon}(g(t))$.[2]

When unemployment satisfies (4), $g(t)$ can be interpreted as an operational time scale for the unemployment rate. To see this, let $\xi(s)$ denote the continuous-time latent process corresponding to the unemployment

[1] See Pissarides (1986, pp. 511–12) for a discussion of "redundancies by wastage" in the United Kingdom.

[2] To verify that (4) is the integral equation corresponding to (3), differentiate (4) and substitute (4) into the resultant expression to obtain $du/dt=\dot{g}\{\alpha_0+\tilde{\epsilon}[g(t)]\}-\theta\dot{g}(t)u(t)$, where $\dot{g}(t)=dg(t)/dt$. The result (3) then obtains by rearranging this expression, using $\dot{g}(t)=\rho(t)$ and $\tilde{\epsilon}[g(t)]=\epsilon(t)$.

rate, defined on the operational time scale s, let $s=g(t)$, and let $u(t)=\xi[g(t)]$. Suppose that $\xi(s)$ has the first-order operational time autoregressive representation

$$\frac{d\xi(s)}{ds}=\alpha_0-\theta\xi(s)+\sigma\tilde{\epsilon}(s), \quad s=g(t). \tag{5}$$

Then $\xi(s)$ satisfies the integral equation

$$\xi(s)=e^{-\theta(s-s')}\xi(s')+\int_{s'}^{s}[\alpha_0+\sigma\tilde{\epsilon}(r)]e^{-\theta(s-r)}\,dr. \tag{6}$$

Letting $s'=-\infty$ and $s=g(t)$, and using the assumptions that $u(t)=\xi[g(t)]$ and $\theta>0$, one obtains from (6) that $u(t)$ obeys (4). Because the integral equations corresponding to (3) and (5) are the same, the nonlinear differential equation (3) can be thought of as being the observational time (t) representation of an unemployment process that satisfies a linear first-order differential equation in continuous operational time (s), where the two time scales are linked by $s=g(t)$.[3]

In the first-order case considered here, the integral equation (6) can be rewritten to yield a discrete-time first-order autoregressive representation for the unemployment rate. The nonlinear time-scale transformation introduces nonlinearities in the discrete-time process because the autoregressive coefficient and the variance of the innovation depend on $g(t)$. Let u_t denote the discrete observational time unemployment rate defined on $t=0,1,2,\ldots,T$. Substituting $s=g(t)$ and $s'=g(t-1)$ into (6), one obtains

$$u_t=\delta_t+\gamma_t u_{t-1}+\nu_t, \tag{7}$$

where

$$\delta_t=\frac{\alpha_0}{\theta}[1-e^{-\theta\Delta g(t)}]$$

$$\gamma_t=e^{-\theta\Delta g(t)}$$

$$\nu_t=\sigma\int_{g(t-1)}^{g(t)}e^{-\theta[g(t)-r]}\tilde{\epsilon}(r)\,dr,$$

where $\Delta g(t)\equiv g(t)-g(t-1)=\int_{t-1}^{t}\rho(\tau)\,d\tau$. By assumption, $\tilde{\epsilon}(t)$ has mean zero and unit variance, so the moments of ν_t are

$$E\nu_t=0, \qquad E\nu_t^2=\frac{\sigma^2}{2\theta}[1-e^{-2\theta\Delta g(t)}].$$

[3] In the terminology of subordinated stochastic processes, $g(t)$ is termed a directing process. See Clark (1973) for a discussion when (unlike here) the subordinated process has independent increments.

The continuous-time model (5) and its discrete-time counterpart (7) are first-order univariate time-deformation models. Their properties and several examples are discussed in Stock (1987, 1988). Two features of the model are worth noting here in relation to the previous discussion concerning hysteresis and time-varying transition probabilities.

First, according to (7), the discrete-time unemployment process has a time-varying conditional mean and is heteroscedastic. The extent to which unemployment is persistent on average depends on the root θ in the continuous-time autoregression: A value of θ close to zero implies high average persistence. Relating this to the flows motivation (1) and (2), at low flow rates (i.e., small values of α and β) labor-force turnover in a typical quarter is limited, and the serial dependence in the unemployment rate is high.

Second, the increments of the time transformation are integrals of the deterministic component of the flow rates in (2), that is,

$$\Delta g(t) = \int_{t-1}^{t} \rho(\tau)\, d\tau.$$

When $\Delta g(t)$ is relatively large, many units of operational time occur during a single unit of observational time. In this case, the dependence of u_t on u_{t-1} is diminished, and the variance of v_t is increased. This formalizes the notion that, in the search-theoretic interpretation of (2), the transition probabilities can be thought of as time-scale parameters: At more intensive levels of search (or during times that search is more productive) and under assumption (2), the unemployment rate literally evolves more rapidly.

The assumptions embodied in (7) will be relaxed in two important ways in the subsequent empirical investigation. First, it has been assumed so far that $\rho(t)$ is a deterministic function of time. If, however, the time-scale transformation is allowed to depend on the past level of unemployment, then the serial correlation in unemployment depends on its own lagged level, as suggested in the discussion of hysteresis in Section 1. Alternatively, the search-theoretic motivation for (1) and (2) suggests that $\rho(t)$ depends on economic variables related to decisions to enter and to leave employment, such as the distribution of reservation wages among job seekers, their misperceptions of real wages, and some measures of job availability such as the vacancy–unemployment ratio. This suggests considering time-scale transformations that depend on a vector of predetermined variables z_{t-1} such that $\epsilon(s)$ is independent of z_{t-1} for $s > t-1$. Specifically, in the empirical analysis we adopt the parametric form used in Stock (1987, 1988),

$$\rho(t) = \frac{\exp(c'z_{t-1})}{T^{-1}\sum_t \exp(c'z_{t-1})}. \tag{8}$$

Note that (8) normalizes $\Delta g(t)$ to be one on average, and ensures that $\Delta g(t) > 0$ for finite c and z_{t-1}.

Second, the model implies that u_t obeys a nonlinear first-order autoregressive process. However, the empirical evidence to be presented suggests that unemployment is better described as a second-order process. Thus the estimated time-deformation models will be based on both first- and second-order continuous-time autoregressions subject to time deformation with time-scale transformations given by (8).[4]

3 Linear time-series properties of unemployment

We first characterize the properties of the U.S. and U.K. unemployment rates in the context of linear models. Both series are quarterly, with observations made on the third month of every quarter.[5]

The unit root and time-trend properties of the unemployment data are examined in Table 1. The first set of statistics tests the hypothesis that unemployment contains a unit root, perhaps with drift, against the alternative that it is stationary, either with a constant mean (the $\hat{\tau}_\mu$ tests) or around a deterministic time trend (the $\hat{\tau}_\tau$ tests). The second set of statistics tests for the presence of a deterministic time trend or a drift.

In the U.S. data, the hypothesis of a unit root is generally rejected at the 10% level in favor of an autoregressive specification in which unemployment is stationary around a time trend. However, in the U.K. data there is little evidence against the unit-root hypothesis using any of the tests. Because of the apparent presence of the unit root in British unemployment, the t-statistic on time in a regression of unemployment on a constant, time, and lag of unemployment has a nonstandard distribution (Fuller, 1976; Sims, Stock, and Watson, in press). However, the test for a drift in first differences of U.K. unemployment is not significant.

[4] By assuming that job quitters must wait one period before becoming eligible for a new job offer, Wright (1986) obtains a second-order difference equation for unemployment that he interprets in terms of search theory. His model is of interest here because, as in (1), its parameters depend on transition probabilities that vary over time. The time dependence he analyzes arises because of agents' misperceptions of changes in the nominal wage. He provides no empirical evidence addressing these implied nonlinearities, however, and the evidence in Bjorklund and Holmlund (1981) raises serious questions about Wright's emphasis on nominal wage misperceptions as a primary source of persistence in unemployment.

[5] The U.S. data were obtained from the Citibase data base, and the U.K. data were obtained from OECD *Main Economic Indicators*.

Table 1. *Specification tests in discrete-time autoregressive models of unemployment*

Statistic[a]	United States 1951:1–1985:3[b]		United Kingdom 1961:2–1986:4[b]	
r_1	0.958		0.979	
$\hat{\tau}_\mu$, unit-root test	−2.58[c]	−2.46	−1.10	−0.90
$\hat{\tau}_\tau$, unit-root test	−3.63[d]	−3.64[d]	−2.69	−2.51
t-Statistic on time trend in u_t	2.50[d]	2.63[d]	2.44[d]	2.34[d]
t-Statistic on drift in Δu_t	0.30	0.29	1.01	1.13

[a] The r_1 statistic denotes the first sample autocorrelation. The $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ statistics testing for a unit root are described in Dickey and Fuller (1979) and tabulated in Fuller (1976). The usual asymptotic critical values were used to evaluate the significance of the t tests for time trends.
[b] 1951:1 denotes the first quarter of 1951, etc.
[c] Significant at the 10% level.
[d] Significant at the 5% level.

The results in Table 1 confirm the evidence of Blanchard and Summers (1986a) from annual data that both series are highly persistent in the sense of having large autoregressive coefficients. Indeed, it is tempting to describe this persistence by suggesting that unemployment is generated by a linear model with a root of exactly one. But such a model provides an unsatisfactory characterization of the process in the long run, because it predicts that unemployment eventually will wander outside the range of 0–100%. One argument used to justify the application of unit-root models to bounded processes is that, in certain applications, they might provide useful locally linear approximations to a globally nonlinear process. However, the informal descriptions of hysteresis as state-dependent second moments suggest that the features of unemployment of particular economic interest are precisely its departures from this linear approximation. This observation, plus the flows analysis of Section 2, suggests taking a closer look at the nonlinear properties of the unemployment rate.

4 Time-deformation models of unemployment

Various time-deformation models of U.S. and British unemployment were estimated using the parametric time-scale transformation (8), where z_{t-1} was a vector of predetermined variables. The parameters were estimated by maximum likelihood under the assumption that the innovation is gaussian. Evaluation of the likelihood involves two technical difficulties. First,

Table 2. *Time-deformation models of U.S. unemployment, 1951:1–1985:3*[a]

Model	Roots	Estimated coefficients c,[b] where z_{t-1} is								Log likelihood, $\mathfrak{L}$	Likelihood ratio statistic, LR[c]
		u_{t-1}	Δu_{t-1}	$\Delta^2 u_{t-1}$	Δu^+_{t-1}	Δu^+_{t-2}	$(v/u)_{t-1}$	$\Delta(v/u)_{t-1}$	$\Delta^2(v/u)_{t-1}$		
AR(1) operational time process											
1	−0.0036	–	–	–	–	–	–	–	–	−89.599	–
2	−0.0289	0.13 (0.07)	0.88 (0.31)	−0.02 (0.32)	–	–	–	–	–	−80.554	18.09[d]
3	−0.0681	0.10 (0.07)	−0.68 (0.73)	−0.62 (0.54)	4.23 (1.48)	−0.70 (0.86)	–	–	–	−72.394	34.41[d]
4	−0.0500	–	–	–	–	–	−0.50 (0.02)	−0.38 (0.08)	0.16 (0.08)	−65.681	47.84[d]
5	−0.149	0.08 (0.08)	−3.05 (0.67)	1.68 (0.52)	–	–	−0.03 (0.02)	−0.98 (0.14)	0.53 (0.12)	−54.988	69.22[d]
AR(2) operational time process											
6	−0.075, −1.40	–	–	–	–	–	–	–	–	−74.648	–
7	−0.067, −1.46	0.02 (0.03)	0.39 (0.16)	0.07 (0.14)	–	–	–	–	–	−68.792	11.71[d]
8	−0.063, −2.53	0.02 (0.04)	−0.33 (0.36)	0.12 (0.30)	1.85 (0.92)	0.28 (0.46)	–	–	–	−65.555	18.19[d]
9	−0.054, −2.29	–	–	–	–	–	−0.02 (0.01)	−0.19 (0.05)	0.06 (0.04)	−59.269	30.76[d]
10	−0.066, −2.55	−0.01 (0.04)	−1.42 (0.38)	0.89 (0.27)	–	–	−0.02 (0.01)	−0.44 (0.08)	0.23 (0.06)	−52.148	45.00[d]

although the model being estimated is defined in continuous time, the data are collected in discrete time. Second, the nonlinear relation between operational and calendar time implies that data sampled at constant intervals in calendar time are effectively sampled at irregular intervals in operational time. Thus the econometric problem is equivalent to estimating the parameters of a continuous-time process sampled at irregular intervals, where the sampling interval is determined by the time-scale transformation which in turn contains unknown parameters. These difficulties are handled by using the Kalman filter algorithm developed in Stock (1988).

The time-scale variables z_{t-1} were chosen to examine parsimoniously the hysteresis hypothesis discussed in Section 1 and the flows motivation provided in Section 2. The hysteresis hypothesis suggests that the persistence of unemployment is itself state-dependent in the sense that the economy might have a tendency to get stuck at high levels of unemployment. Thus u_{t-1} was included as an explanatory variable in the time scale. Related hypotheses are that the persistence in the process might depend on whether the unemployment rate is increasing or decreasing or on whether the unemployment rate is accelerating or decelerating. Thus Δu_{t-1} and $\Delta^2 u_{t-1} \equiv \Delta u_{t-1} - \Delta u_{t-2}$ were included as elements of z_{t-1} as well. Finally, the persistence of the process might depend on the change in unemployment, but this dependence might differ if unemployment has been increasing rather than decreasing. Consequently, Δu_{t-1}^{+} and Δu_{t-2}^{+} have also been included, where $\Delta u_{t-1}^{+} = \Delta u_{t-1}$ if $\Delta u_{t-1} \geq 0$ and $=0$ otherwise.

The estimation results for the U.S. data are summarized in Table 2. The theoretical development in Section 2 suggests examining specifications with a first-order autoregressive process in operational time; the results are presented in the first half of the table. Model 1 has no time-deformation effects and so is simply a first-order continuous-time autoregression estimated with evenly spaced discrete observations. Models 2 and 3 examine the hysteresis hypothesis by fitting univariate time-deformation models using lagged unemployment. Both models exhibit statistically significant time-deformation effects based on the likelihood ratio statistic. In the models based on u_{t-1}, Δu_{t-1}, and $\Delta^2 u_{t-1}$, the coefficient on u_{t-1} is positive, indicating a relative increase in the operational time scale at high rates of unemployment, corresponding to a relative *decline* in the persistence of the process at high rates (although this effect is quantitatively slight and, in the case of model 3, statistically insignificant). In particular,

Notes to Table 2

[a] Time transformation is $\Delta g(t) = \exp(c'z_{t-1})/[T^{-1}\sum \exp(c'z_{t-1})]$.

[b] Standard errors (in parentheses) were computed using numerical derivatives.

[c] Tests hypothesis that all time-deformation coefficients are zero.

[d] Significant at the 1% level.

models 2 and 3 provide no evidence that persistence *increases* at high rates of unemployment. In contrast, $\Delta g(t)$ rises substantially when unemployment is increasing. Comparing models 2 and 3, the likelihood ratio statistic indicates that the absolute value of Δu_{t-1} (and its lag) enter the time-scale transformations significantly at the 1% level.

The development in Section 2 also suggests considering variables commonly treated as proximate determinants of flows into and out of unemployment. Thus models 4 and 5 have time-scale transformations based on the vacancy–unemployment ratio (see, e.g., Pissarides, 1985) and its first and second difference. (The Conference Board index of help-wanted advertising is used as the vacancies series.) These models exhibit dramatic time-deformation effects. Both models suggest that an increasing vacancy–unemployment ratio corresponds to a decline in $\Delta g(t)$ and an associated increase in the persistence of unemployment.[6]

Although the discussion of Section 2 was for a first-order model, the restriction that the true model be first order need not be satisfied in the data. The models were therefore reestimated assuming a second-order autoregressive latent process; the results are reported in the second half of the table. In every case the hypothesis that the model is first order can be rejected at the 5% level using the usual likelihood ratio statistics, though the test statistics are substantially smaller for the time-deformation models than for the models with no time deformation. Despite these rejections of the AR(1) specification, the estimated time-scale coefficients are broadly similar for comparable models across the two panels. Additional evidence of the similarity of the estimated transformations is provided by the contemporaneous correlations among them reported in Table 3, with 15 of the 28 correlations exceeding 0.75.[7] The high correlations in comparable AR(1) and AR(2) models provide support for a loose interpretation

[6] Using monthly data, Bjorklund and Holmlund (1981) found that vacancies entered significantly into transition probability regressions. The results in Table 2 are consistent with their findings because the v/u ratio is presumed to enter through the transition probabilities α and β.

[7] AR(3) and AR(4) continuous-time latent processes were also estimated. These models typically did not converge, with the most negative root tending toward $-\infty$. This indicates overparameterization of the continuous-time AR models because a continuous-time root of $-\infty$ corresponds to a discrete-time root of 0. As an additional check of the robustness of the estimated time transformations, we reestimated the models in Table 2 including a linear time trend in continuous operational time, though such a specification is largely inconsistent with the spirit of this exercise, which is to model the apparent persistence of high and increasing unemployment using a stable stochastic model. The time transformations estimated with a time trend in the latent process are qualitatively very similar to those reported in Table 2, though in most cases the hypothesis that the time trend is zero can be rejected at the 5% level. These and other unreported results are available from the author upon request.

Table 3. *Correlations of estimated time-scale transformations, United States, 1951:1–1985:3* [a]

Model	2	3	4	5	7	8	9	10
2	1	0.70	0.63	0.31	0.93	0.90	0.92	0.70
3		1	0.79	0.56	0.49	0.92	0.69	0.66
4			1	0.87	0.52	0.81	0.82	0.91
5				1	0.25	0.50	0.56	0.84
7					1	0.77	0.90	0.69
8						1	0.89	0.76
9							1	0.90
10								1

[a] Correlations among $\Delta g(t)$ computed using the estimated models in Table 2.

of these time-scale transformations as changing transition probabilities, as suggested by the development in Section 2, although the test statistics indicate that this interpretation is not strictly warranted.

In summary, two conclusions emerge from Table 2. First, in time transformations based solely on the lagged unemployment rate or its differences, persistence is seen to depend more on the change in unemployment than on the level itself. That is, when U.S. unemployment is stable, it appears to be highly persistent; but a past increase in unemployment is associated with a substantial reduction in its dependence. This suggests that unemployment becomes stuck at a certain level, with high persistence and low innovation variance. However, a large positive innovation is associated with a subsequent reduced dependence and greater innovation variance, that is, with the process becoming unstuck until it moves to a new level with a new and relatively slow operational time scale.

This finding might initially seem unsurprising. In a linear model with positive serial correlation in first differences, a large change in unemployment in one quarter would lead one to forecast a large change in the next. However, the time-deformation results imply something quite different: the greater the increase in one quarter, the *less predictable* is the change in the next. Under a linear specification, this predictability (i.e., serial correlation and heteroscedasticity) is time invariant whereas the test statistics in the final column of Table 2 reject this hypothesis in favor of time deformation.

These results are based on the asymptotic approximation to the likelihood ratio test statistic. To evaluate the adequacy of this approximation in samples of moderate size, a Monte Carlo experiment was performed

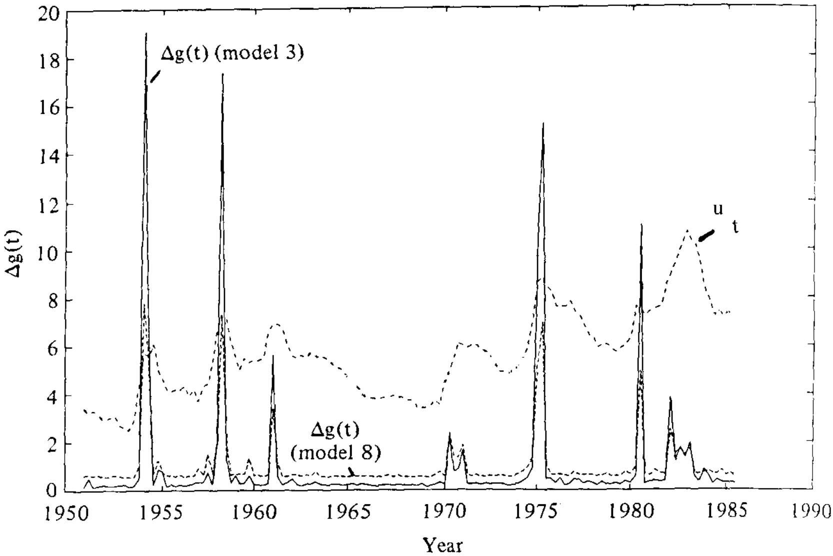

Figure 1. U.S. unemployment rate and the estimated change in its operational time scale. The estimates of $\Delta g(t)$ are from models 3 and 8 in Table 2.

under the null hypothesis of no time deformation. The results, presented in the appendix, support this application of the asymptotic theory. This reinforces the conclusion that the U.S. unemployment rate exhibits substantial nonlinearities associated with past changes in unemployment.

To gain further insight into the estimated nonlinearity in the unemployment rate, the changes in the time scales estimated using models 3 and 8 have been plotted, along with the unemployment rate, in Figure 1. The spikes in Figure 1 indicate periods of markedly large $\Delta g(t)$, corresponding to sharp drops in the dependence of the process and increases in the conditional discrete time innovation variance. As suggested by the point estimates, these spikes correspond to periods of previously increasing unemployment; this pattern is apparent in both the time transformations plotted, though the magnitudes of the spikes differ. Although they are not plotted here, the other estimated time transformations in Table 2 have the same qualitative features evident in Figure 1, with large spikes during periods of increasing and accelerating unemployment.

Do the same nonlinearities appear in the U.K. data? The estimation results using the U.K. unemployment rate are presented in Table 4. In the

Table 4. *Time-deformation models of U.K. unemployment, 1961:2–1986:4*[a]

Model	Roots	Estimated coefficients c,[b] where z_{t-1} is u_{t-1}	Δu_{t-1}	$\Delta^2 u_{t-1}$	Δu^+_{t-1}	Δu^+_{t-2}	Log likelihood, $\mathcal{L}$	Likelihood ratio statistic, LR[c]
AR(1) operational time process								
1	−0.002	–	–	–	–	–	−27.900	–
2	−0.002	0.11 (0.05)	2.12 (0.49)	−0.15 (0.80)	–	–	−6.145	43.51[d]
3	−0.003	0.09 (0.05)	−2.35 (1.84)	−0.30 (1.49)	6.88 (1.94)	−0.78 (1.87)	−0.220	55.36[d]
AR(2) operational time process								
4	−0.016, −0.432	–	–	–	–	–	16.337	–
5	−0.013, −0.423	0.04 (0.02)	0.036 (0.25)	−0.40 (0.27)	–	–	26.871	21.07[d]
6	−0.015, −0.390	0.05 (0.02)	1.10 (0.70)	−0.85 (0.46)	−0.44 (0.85)	−0.57 (0.54)	27.527	22.38[d]

[a] Time transformation is $\Delta g(t) = \exp(c'z_{t-1})/[T^{-1}\sum \exp(c'z_{t-1})]$.
[b] Standard errors (in parentheses) were computed using numerical derivatives.
[c] Tests hypothesis that all time-deformation coefficients are zero.
[d] Significant at the 1% level.

Table 5. *Correlations of estimated time-scale transformations, United Kingdom, 1961:2–1986:4*[a]

Model	2	3	5	6
2	1	0.90	0.74	0.71
3		1	0.48	0.45
4			1	0.99
5				1

[a] Correlations among $\Delta g(t)$ computed using the estimated models in Table 4.

absence of a measure of vacancies analogous to that used for the U.S. data, only time-deformation models based on lags and lagged differences of unemployment were estimated. All models indicate statistically significant time-deformation effects. However, the evidence against the first-order specification in Table 4 is overwhelming, with χ_1^2 likelihood ratio statistics all exceeding 50. These test results, the small second continuous-time roots reported in the second half of the table, and the differences in the time-scale coefficients in corresponding AR(1) and AR(2) models therefore suggest that the first-order model discussed in Section 2 is sharply inconsistent with the empirical evidence. In addition, the correlations in Table 5 indicate only a modest relation between the two sets of estimated time transformations. The remaining discussion therefore focuses on the estimated second-order models.

An interesting feature of the results is the similarity of the coefficients on u_{t-1} and Δu_{t-1} in model 5 of Table 4 for the United Kingdom to the corresponding coefficients for the U.S. time-deformation models reported in model 7 of Table 2. The primary differences between these two estimated models is the relatively small second root in the British continuous-time model and the opposite signs of the coefficients on $\Delta^2 u_{t-1}$; although an acceleration of unemployment in the United States corresponds to a slight decrease in persistence, a similar acceleration in the United Kingdom corresponds to an increase in persistence.

This difference is reflected in the plots of the respective time transformations. The estimated time transformation from model 6 is plotted in Figure 2; this transformation involves the same time-scale variables as the U.S. transformations plotted in Figure 1. As suggested by the estimated coefficients, $\Delta g(t)$ is greatest (and persistence smallest) at high levels of unemployment and with decelerating unemployment. Accordingly, the

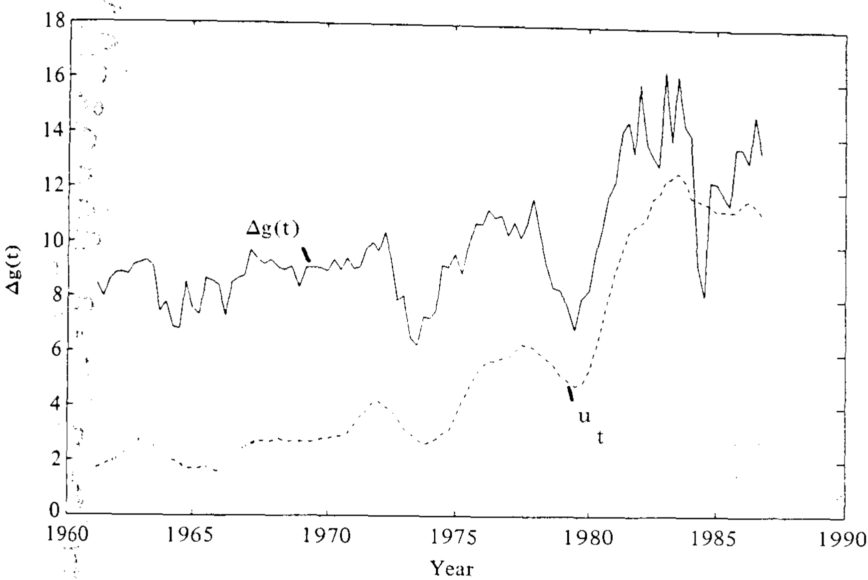

Figure 2. U.K. unemployment rate and the estimated change in its operational time scale. The estimate of $\Delta g(t)$ is from model 6 in Table 4. The plotted values of the time-scale transformation have been scaled up by a factor of ten; the sample average of $\Delta g(t)$ is 1 by construction.

lowest dependence is seen to occur in the 1980s. This contrasts with the view that U.K. unemployment in the early 1980s has exhibited unusually high persistence. Indeed, the opposite appears to be true, at least according to Figure 2.

5 Conclusions

The empirical results of Section 4 indicate substantial and statistically significant nonlinearities in the unemployment process. These findings can be interpreted as indicating a nonlinear relation between the operational and observational time scales of unemployment, a view motivated partly because the time-deformation model captures the qualitative features of state dependence that characterize hysteresis and partly by considering the model of unemployment flows presented in Section 2. This interpretation warrants two caveats. First, although the time-deformation model captures the intuitive notion that unemployment might evolve on a time scale other than calendar time, it imposes a restrictive parameterization on the nonlinearities; other models might detect different nonlinear features in the data. Second, using U.S. data, although the vacancy–unemployment ratio significantly enters the time-scale transformation (as predicted by the search arguments in Section 2), this does not support any particular formulation of search theory or the associated welfare implications. In particular, workers and the unemployed might engage in optimal search, but the set of available jobs could be influenced by macroeconomic policy; this is consistent with the discussions in Blanchard and Summers (1986a, b, c), Pissarides (1986), Gottfries and Horn (1986), and Jones (1987).

The empirical results characterize nonlinear patterns in the unemployment series using a variety of proxies for the time-scale transformation variables. Narrowly interpreted, the results indicate that both U.S. and U.K. unemployment exhibit reduced persistence and greater conditional heteroscedasticity at high than at low rates of unemployment and, more importantly, during times of increasing unemployment.

Interpreting these results somewhat more broadly, these models provide little support for the proposition that unemployment exhibits greater dependence at high than at low levels. Rather, the picture that emerges is: If unemployment has been stable then its dependence is increased. In contrast, if unemployment has been *rising* in the recent past then its dependence is reduced, and the error associated with forecasts of future unemployment increases. This empirical result resembles descriptions of hysteresis in unemployment as shifts between multiple persistent equilibria.

Appendix: A Monte Carlo evaluation of the time deformation MLE

This appendix describes a small Monte Carlo experiment designed to investigate the quality of the asymptotic χ^2 approximation to the distribution of the likelihood ratio statistic testing for the presence of time deformation, under the null hypothesis that there is no time deformation. The data were generated under the maintained assumption that the univariate process obeys a first-order linear stochastic differential equation in continuous operational time.

Two experiments were performed. In both, the data were assumed to obey the stochastic differential equation

$$dX(s) = [\mu - \theta X(s)]\, ds + \sigma d\zeta(s), \quad \mu = 0, \ \theta = 0.693, \ \sigma = 1.848,$$

where $\zeta(s)$ is standard Brownian motion and there is no time deformation, so that $s = t$. Thus, by using (7) with $\Delta g(t) = 1$, the pseudodata were generated by the discrete-time AR(1) process $X_t = 0.5X_{t-1} + \nu_t$, where

ν_t is independently distributed $N(0,1)$. In the first experiment, the time-scale transformation was of the form (8) and depended on a single lag of x_t; that is,

$$k=1: \quad \Delta g_1(t) = \frac{\exp(cx_{t-1})}{T^{-1}\sum_t \exp(cx_{t-1})}.$$

In the second experiment, the time transformation also depended on the change in x_{t-1}; that is,

$$k=2: \quad \Delta g_2(t) = \frac{\exp(c_1x_{t-1}+c_2\Delta x_{t-1})}{T^{-1}\sum_t \exp(c_1x_{t-1}+c_2\Delta x_{t-1})}.$$

The unknown parameters are (μ, θ, σ, c) for $k=1$ and $(\mu, \theta, \sigma, c_1, c_2)$ for $k=2$. In both experiments, the likelihood ratio statistic was evaluated by estimating the model subject to the constraint that $c=0$ (or $c_1=c_2=0$) and then not subject to this constraint. Estimation was by maximum likelihood using the modified Kalman filter in Stock (1988). Initial values for the maximization (performed in FORTRAN using the Davidon–Fletcher–Powell maximization algorithm) were $(\mu, \theta, \sigma, c)_0 = (0, 0.5, 0.5, 0)$ for $k=1$ and $(\mu, \theta, \sigma, c_1, c_2)_0 = (0, 0.5, 0.5, 0, 0)$ for $k=2$. Both experiments involved 100 replications with a sample size of 100, requiring a total of 32 hours on an IBM AT. All 400 likelihood maximizations (100 each for the constrained and unconstrained cases with $k=1$ and $k=2$) converged.

As Table 6 indicates, in the Monte Carlo simulations the nominal sizes of the tests based on the likelihood ratio statistic are close to the asymptotic significance level. Although there is some tendency for the size to exceed the level, the standard error associated with these estimates of the size is large (approximately 0.03 for the 10% level entries). However, the results provide no evidence of a dramatic failure of the asymptotic approximation (or the optimization routine), nor is there noticeable deterioration of the asymptotic approximation when two time-deformation parameters are estimated rather than one.

Quantiles of the Monte Carlo distribution of various estimators are presented in Table 7. The estimated autoregressive coefficient is skewed away from zero, being median- but not mean-unbiased in both the restricted and the unrestricted case. There is no noticeable deterioration of the distribution of $\hat{\theta}$ moving from the restricted to the unrestricted case. In addition, the time-deformation parameters appear to be symmetrically distributed around zero in both experiments. Although the range of the estimated time-deformation parameters is large, this does not imply that these parameters would be estimated imprecisely were time deformation in fact present, an issue not addressed in these experiments.

Table 6. *Nominal sizes of likelihood ratio tests for time deformation using asymptotic critical values*[a,b]

	Size[c]	
Level (α)	$k=1$	$k=2$
0.05	0.06	0.08
0.10	0.15	0.17
0.20	0.25	0.21
0.30	0.37	0.28
0.50	0.68	0.50
0.70	0.76	0.71
0.90	0.92	0.89

[a] The critical values for each significance level were taken from tables of the asymptotic $\chi^2_{k;\alpha}$ distribution.
[b] Sample size = 100.
[c] Entries are based on 100 Monte Carlo replications as described in the text. For example, the $k=1$, $\alpha=.05$ entry reports that six of the 100 LR statistics drawn in the $k=1$ experiment indicate rejection of the null of $c=0$ (i.e., of no time deformation) based on the χ^2_1 5% critical value of 3.84.

Table 7. *Percentiles and summary statistics of selected estimators*[a]

Monte Carlo percentile and mean	Quantiles of estimated coefficients when k time-deformation coefficients are estimated[b] $k=0$	$k=1$		$k=2$		
	$-\hat{\theta}$	$-\hat{\theta}$	$\hat{c}$	$-\hat{\theta}$	$\hat{c}_1$	$\hat{c}_2$
10%	−1.06	−1.26	−0.24	−1.15	−0.39	−0.31
30%	−0.86	−0.87	−0.10	−0.86	−0.12	−0.12
50%	−0.72	−0.69	0.00	−0.75	−0.01	−0.02
70%	−0.61	−0.62	0.07	−0.64	0.08	0.13
90%	−0.55	−0.52	0.19	−0.52	0.26	0.37
Mean	−0.85	−0.95	−0.01	−0.83	−0.03	0.01
True value	−0.693	−0.693	0.00	−0.693	0.00	0.00

[a] Sample size = 100.
[b] The restricted estimates in the $k=0$ column were computed imposing the true restriction that there is no time deformation whereas $k=1$ and $k=2$ refer to the number of time-deformation parameters estimated in the unrestricted experiments. Entries in the $k=0$ column are based on 200 Monte Carlo replications (combining the estimates from the $k=1$ and $k=2$ experiments); all other entries are based on 100 Monte Carlo replications.

These experiments were performed under conditions likely to yield good performance of the optimization routine and of the asymptotic theory, that is, a first-order gaussian process with few parameters to estimate and starting values that are not too far from the true values. Nevertheless, the results provide no evidence of an important failure of the algorithm or the asymptotic approximations.

REFERENCES

Bjorklund, A., and Holmlund, B. (1981), "The duration of unemployment and unexpected inflation: An empirical analysis," *American Economic Review*, 71(1), 121–31.

Blanchard, O. J., and Summers, L. H. (1986a), "Hysteresis and the European unemployment problem," *NBER Macroeconomics Annual*, 15–78.

(1986b), "Fiscal increasing returns, hysteresis, real wages and unemployment," National Bureau of Economic Research Working Paper No. 2034.

(1986c), "Hysteresis in unemployment," National Bureau of Economic Research Working Paper No. 2035.

Brock, W. A., and Sayers, C. L. (1986), "Is the business cycle characterized by deterministic chaos?" Working Paper 8617, Social Systems Research Institute, University of Wisconsin, Madison.

Clark, P. K. (1973), "A subordinated stochastic process model with finite variance for speculative prices," *Econometrica*, 41, 135–56.

DeLong, J. B., and Summers, L. H. (1986), "Are business cycles symmetrical?" in *The American Business Cycle*, ed. by R. J. Gordon, Chicago: University of Chicago Press, 166–79.

Diamond, P. A. (1982), "Aggregate demand management in search equilibrium," *Journal of Political Economy*, 90(5), 881–94.

Dickey, D. A., and Fuller, W. A. (1979), "Distribution of the estimators for autoregressive time series with a unit root," *Journal of the American Statistical Association*, 74(366), 427–31.

Fuller, W. A. (1976), *Introduction to Statistical Time Series*, New York: Wiley.

Gottfries, N., and Horn, N. (1986), "Wage formation and the persistence of unemployment," Institute for International Economic Studies Working Paper No. 347, University of Stockholm.

Jones, S. R. G. (1987), "The relationship between unemployment spells and reservation wages as a test of search theory," manuscript, Institute for Advanced Study, Princeton.

Neftci, S. N. (1984), "Are economic time series asymmetric over the business cycle?" *Journal of Political Economy*, 92, 307–28.

Pissarides, C. A. (1985), "Short-run equilibrium dynamics of unemployment, vacancies, and real wages," *American Economic Review*, 75(4), 676–90.

(1986), "Unemployment," *Economic Policy*, 3, 501–41.

Sims, C. A., Stock, J. H., and Watson, M. W. (in press), "Inference in linear time series models with some unit roots," *Econometrica*.

Stock, J. H. (1987), "Measuring business cycle time," *Journal of Political Economy*, 95(6), 1240–61.

(1988), "Estimating continuous time processes subject to time deformation: An application to postwar GNP," *Journal of the American Statistical Association*, 83, 77–85.

Tobin, J. (1980), "Stabilization policy ten years after," *Brookings Papers on Economic Activity,* 1, 19–71.

Wright, R. (1986), "Job search and cyclical unemployment," *Journal of Political Economy,* 94(1), 38–55.